Making School Mathematics Functional

a stool needs three legs

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Canadian Mathematics Education Forum
Vancouver, May 2009
Structure

- Mathematics that is functional
- Performance goals in Mathematics
- The three legs of systemic change
- Task design: issues, strategies, tactics
- Teaching and teaching materials
- Supporting professional development
The Shell Centre Team

- Malcolm Swan, Daniel Pead, Rita Crust, Alan Bell, HB, with many associates
- Tool design engineers doing engineering research in education, ie design and development of:
  - teaching materials and processes
  - assessment tasks
  - professional development materials and processes
  - tools and strategies for system change
  with some associated ‘insight research’
- Based in the University of Nottingham School of Education
- Works with many others, notably Berkeley, Michigan State, and school systems in UK and US
- Contact: Hugh.Burkhardt@nottingham.ac.uk
  www.mathshell.com
Functional Mathematics

Non-specialist adults, if they are taught how, benefit from using mathematics in their everyday lives to better understand the world they live in, and to make better decisions.

“The sophisticated use of, often elementary, mathematics” also called mathematical literacy (ML), quantitative literacy, numeracy ...

Post-age-11 mathematics is non-functional for most people
Max has just received this email

From: A. Crook
To: B. Careful

Do you want to get rich quick?
Just follow the instructions carefully below and you may never need to work again:

1. Below there are 8 names and addresses. Send $5 to the name at the top of this list.
2. Delete that name and add your own name and address at the bottom of the list.
3. Send this email to 5 new friends.
“PONZI” PYRAMID SCHEMES

- If that process goes as planned, how much money would be sent to Max?
- What could possibly go wrong?
- Why do they make Ponzi schemes like this illegal?

builds understanding of standard scam – sees the power of exponential growth, and why it can’t go on for ever
Making a case

The spreadsheet contains 2 sets of reaction times, 100 each for Joe and Maria.

- Using this data, construct two arguments:
  - A: that Joe is quicker than Maria
  - B: that Maria is quicker than Joe

builds understanding, and intelligent scepticism, of how political and marketing data is used – uses different summative measures on the same data
The modelling process

The real world

Problem
Formulate
Solve
Interpret
Validate
Report

Mathematics
Dysfunctional math curricula

The real world

Solve

Mathematics
What does ML involve?

- “The sophisticated use of, often elementary, mathematics”
- All key aspects of ‘doing mathematics’
  - Beliefs
  - Strategies
  - Techniques
  - Metacognition
  - Control
- “The Few Year Gap” between imitation and autonomy
cf Specialist Mathematics

SM provides the mathematical toolkit for further study in socially important fields: engineering, physics, economics, ....

*into which an important minority will go.*

SM shows more of the intellectual excitement of mathematics (cf music)

Here I will focus on functional mathematics because if its:

- Social importance for all
- Motivation for most

Specialist mathematics, done properly, needs all the same things.
Modelling

- Joe buys a six-pack of coke for $3 to share among his friends. How much should he charge for each bottle?

- If it takes 40 minutes to bake 5 potatoes in the oven, how long will it take to bake one potato?

- If King Henry 8th had 6 wives, how many wives had King Henry 4th?
Teaching modelling: some history

- 1960- individual experimental courses
- Scale of implementation, mainly UK and US
  - 1970-90 some UG courses (ICTMAfia)
  - 1990- –ve progress/cosmetic realism
- Now: in some Germany (regions) a coherent move to establish modelling
- England: adopts “functional maths” – meaning unclear
If you drop into 100 randomly chosen mathematics classrooms, will you see modelling? **Unlikely**

Why? Unsolved problem but …
- Broader teaching skills than imitative curriculum
- Mathematics remains inward-looking
- Deep change needs **pressure and support**

**Don’t give up**

Research > large scale practice 25 years
- Penicillin, vacuum cleaner, gene therapy
- Systemic change makes it harder
The three legs of the stool

- Assessment
- Professional development
- Teaching materials

How do we get them balanced?

What kinds of tools and processes do we need to make this happen?
Pressure + Support

- System and culture dependent
- Pressure: good or bad
  - Anglos: high-stakes tests + National Curriculum + inspections
  - *What is it in your province/state/country?*
  - PISA?
- Support
  - Teaching materials
  - Professional development
  - *What is it in your country?*

To work, these must be well-engineered+aligned
Professional development pathway

0. Managing the class
   1. Delivering the textbook
   2. Adding good activities (eg NCTM)
      Many teachers “plateau” here
      For some teachers, this routine expertise then develops into adaptive expertise
      (Hatano, Schoenfeld)

3. Building on where each student is
   Catalyzing and supporting that shift is the core challenge of PD, involves changing:
   1. Knowledge – of math and pedagogy
   2. Orientation – the “classroom contract”
   3. Goals – dimensions of performance
Issues in task design

- The roles of assessment
- Performance goals in Mathematics
- Task design principles
- Task design: issues, strategies, tactics
- Building tests within constraints
Roles of high-stakes assessment

Role A: Measures levels of performance

Role B: exemplifies performance objectives

Role C: determines classroom activity

Standard errors: only consider A rely on correlation

(Paleo-)Psychometrics ignores what is assessed

What design responsibilities do A+B+C imply?
The importance of good tasks

show performance goals in a compact way

Types of mathematical task

- reproduce a learned procedure
  - such ‘exercises’ now dominate
- critique and improve
- plan
- design
- evaluate and recommend
- investigate
- .....
Alison and two friends has planned a cycling trip around Derbyshire on Saturday.

Here is their plan for the day.

Read through the plan and the information sheets (next page).

If you find a mistake, or realise something has been forgotten, write it down and say how they should change the plan.

Meet at Loughborough station at 7.23 am. Buy tickets and then catch the train to Derby. This arrives at 7.51 am.

At Derby, catch the 8.20 am train to Cromford. This arrives at 8.41 am.

Here are the instructions for getting to the Cycle Hire centre:

“Turn left as you come out of Cromford station, walk along by the river and down Mill road. Cross over the A6. Walk up Cromford hill for about 1/2 mile and you will see..
Cycle hire information

<table>
<thead>
<tr>
<th>CYCLE HIRE CENTRE</th>
<th>LOCATION</th>
<th>JAN</th>
<th>FEB</th>
<th>MAR</th>
<th>APR</th>
<th>MAY</th>
<th>JUN</th>
<th>JUL</th>
<th>AUG</th>
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<th>OCT</th>
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</tbody>
</table>

WHERE TO?

Cycles are available at the following cycle hire centres:
- Buxton
- Ashbourne
- Parsley Hay
- Bakewell
- Chatsworth
- Matlock

Average cycling speed is about 8 mph.

WHERE FROM?

Cycles can be hired from the following cycle hire centres:
- Buxton
- Ashbourne
- Parsley Hay
- Bakewell
- Chatsworth
- Matlock

Children's cycle hire:
- Children aged 3 and over can hire a child's cycle free of charge.
- A child's cycle must be returned to the same hire point from where it was borrowed.

For further information, please contact the cycle hire centre.

Contact details:
- Buxton: 01629 739444
- Ashbourne: 01629 739444
- Parsley Hay: 01629 739444
- Bakewell: 01629 739444
- Chatsworth: 01629 739444
- Matlock: 01629 739444
Six people are planning a day out. Six different places have been suggested: Ice rink; Bowling alley; Swimming pool; Zoo; Castle; Snooker hall. They take a vote. Which would be the best place for the trip and why?

<table>
<thead>
<tr>
<th>Name</th>
<th>Choice 1</th>
<th>Choice 2</th>
<th>Choice 3</th>
<th>Choice 4</th>
<th>Choice 5</th>
<th>Choice 6</th>
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<tr>
<td>Sanjay</td>
<td>Ice Rink</td>
<td>Zoo</td>
<td>Bowling</td>
<td>Castle</td>
<td>Snooker</td>
<td>Swimming</td>
</tr>
<tr>
<td>Mike</td>
<td>Ice Rink</td>
<td>Zoo</td>
<td>Bowling</td>
<td>Castle</td>
<td>Snooker</td>
<td>Swimming</td>
</tr>
<tr>
<td>John</td>
<td>Ice Rink</td>
<td>Zoo</td>
<td>Bowling</td>
<td>Castle</td>
<td>Snooker</td>
<td>Swimming</td>
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<tr>
<td>Claire</td>
<td>Ice Rink</td>
<td>Zoo</td>
<td>Bowling</td>
<td>Castle</td>
<td>Snooker</td>
<td>Swimming</td>
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<tr>
<td>Elaine</td>
<td>Ice Rink</td>
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<td>Bowling</td>
<td>Castle</td>
<td>Snooker</td>
<td>Swimming</td>
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<td>Jenny</td>
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<td>Zoo</td>
<td>Bowling</td>
<td>Castle</td>
<td>Snooker</td>
<td>Swimming</td>
</tr>
</tbody>
</table>
In the population as a whole, about 1 baby in 8,000 dies in an unexplained "cot death". The cause or causes are at present unknown.

Three successive babies in one family have died.

The mother is on trial. An expert witness says:

"One cot death is a family tragedy; two is suspicious; three is murder. The odds on three deaths in one family are 64 million to 1"

Discuss the reasoning behind the expert witness' statement, noting any errors, and write an improved version to present to the jury.
I have found it useful to distinguish

A  Action problems – for now
B  Believable problems – for the future
C  Curious problems – for delight
D  Dubious problems (look in any math book)
E  Educational problems – D but OK
Dimensions of performance

- Content: math topics, concepts, skills
- Phases of problem solving/modeling
- Non-routine-ness
- Open-ness: closed, open middle, end
- Goal type: applied power, pure math
- Reasoning length
- Task type
An important distinction

- **Illustrative applications** show standard models
- **Active modelling** of situations you know well, but have not previously analysed, is essential for ML
Task difficulty

Depends on a combination of

- Complexity
- Unfamiliarity
- Technical demand
- Student autonomy

Cannot be reliably predicted, hence trials

“Few year gap” v imitative exercises
Sue and Terry are making dogs and teddy bears.

They have time to make 18 toys, and £60 to spend on materials. Materials for a dog cost £3, materials for a teddy bear cost £4. They sell each dog for £8 and each teddy bear for £10.

How many of each should they make to maximise profit?
Three girls compete to be selected for the regional long jump competition.

Each has six jumps; the results are shown in the table.

Which girl should be selected? Explain your reasoning.

*from TIMSS video study*
Long jump

- They calculated the average jump for each girl!! >> Olga
- The teacher moved on
- There was no discussion of other/appropriate measures – a worthwhile task.

<table>
<thead>
<tr>
<th></th>
<th>Elsa</th>
<th>Ilse</th>
<th>Olga</th>
</tr>
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<tbody>
<tr>
<td>1st</td>
<td>3.25</td>
<td>3.55</td>
<td>3.67</td>
</tr>
<tr>
<td>2nd</td>
<td>3.84</td>
<td>3.99</td>
<td>3.78</td>
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<tr>
<td>3rd</td>
<td>4.10</td>
<td>3.61</td>
<td>3.92</td>
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<tr>
<td>4th</td>
<td>2.95</td>
<td>3.97</td>
<td>3.62</td>
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<tr>
<td>5th</td>
<td>3.66</td>
<td>3.69</td>
<td>3.85</td>
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<tr>
<td>6th</td>
<td>3.86</td>
<td>3.59</td>
<td>3.73</td>
</tr>
</tbody>
</table>
Mathematics uses Computers everywhere…… except in school math

- Computers are valuable tools for:
  - organising data, and thinking – spreadsheets
  - finding information – via the web…
  - simulating real world problems
  - doing + checking messy procedures…….

.. but school implementation is challenging
  - timescale mismatch
  - equity concerns
  - teacher skills

- Modularising may help
Who needs it?

Plan

- Potential secretaries asked to critique and complete the spreadsheet for planning a conference budget

Graduates who “know Excel” don’t create formulas in col E

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>College charges</td>
<td>Delegate s @ £ each</td>
<td>£</td>
<td></td>
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<td>Buffet Supper</td>
<td>30</td>
<td>17.00</td>
<td>0</td>
</tr>
<tr>
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<td>Single En-suite Accommodation</td>
<td>30</td>
<td>40.00</td>
<td>0</td>
</tr>
<tr>
<td>Tuesday</td>
<td>Breakfast</td>
<td>30</td>
<td>8.00</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Morning Coffee</td>
<td>30</td>
<td>1.90</td>
<td>0</td>
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<tr>
<td></td>
<td>Luncheon</td>
<td>30</td>
<td>15.00</td>
<td>0</td>
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<tr>
<td></td>
<td>Afternoon tea</td>
<td>30</td>
<td>1.90</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Dinner served</td>
<td>30</td>
<td>50.00</td>
<td>0</td>
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<tr>
<td></td>
<td>Single En-suite Accommodation</td>
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<td></td>
<td>Plenary Room</td>
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<td>15.77</td>
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<tr>
<td></td>
<td>Breakout rooms</td>
<td>2</td>
<td>85.10</td>
<td>0</td>
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<tr>
<td>Wednesday</td>
<td>Breakfast</td>
<td>30</td>
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<td>Luncheon</td>
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<td>Afternoon tea</td>
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<td></td>
<td>No Dinner</td>
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<td>Single En-suite Accommodation</td>
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<td>Plenary Room</td>
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<td>Breakout rooms</td>
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<td>Thursday</td>
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<td></td>
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</table>
Tree rings

Trees grow by making new wood just under their bark.

Each year, a new layer of wood is added.

When a tree is cut down, you can see these layers as a pattern of rings.

You can tell the age of the tree in years by counting the rings.

Press "Go" and watch the movie.

1. ☑️ The tree in the movie was cut down in the year 2000.

   In what year was it planted?   ?

   In what year did it grow the most?   ?
Sandra and Ian have collected data on different types of bird. They know how fast they fly, how much they weigh and the area of their wings.

They want to know if they can predict how fast a bird will fly from its size and shape.

Now go on to page 2
Teaching Math Literacy

For Mathematical Literacy units so far, it seems:

- All students succeed and enjoy the work
- ML narrows the range of performance
- Many, but not all, teachers can handle this work with just the materials – more with live PD training
- 1 or 2 new three-week units per year is digestible

More research needed, across more exemplar units to warrant such general statements
“Bowland Maths”

- ~20 “case studies, including:
  - Reducing road accidents
  - How risky is life?
  - “You reckon?”
  - Alien invaders
- Professional development
  - 5 module package, activity based
- Assessment

The importance of design + engineering
What do good designers do?

They know how to

- use research results and design skills to
  - improve ‘best practice’
  - tackle new challenges effectively
- pass on their knowledge to
  - other practitioners
  - novices
through their materials.
Educational design principles

Heuristic, phenomenological theory:
- Some based on ‘insight research’, eg
  - active learning
  - constructive
  - build multiple connections
- Others design-based, eg
  - role shifting
  - cognitive conflict
  - student ‘ownership’

Design theory is not often discussed in enough detail to be useful
Design beyond just principles

Design brilliance is more than these:
- ‘Surprises’ that are clearly ‘right’
- Handling complexity simply
- Controlled innovation
- Balance in all aspects

We know it when we see it – iPod, ...
What development skills are needed?

- The team needs:
  - Systematic methods of observation
  - Interview skills
  - Protocols related to the design goals
  - Methods for analysing observation reports, student work, interviews

- Design skill in using this rich feedback systematically to improve the materials.

  *i.e. as products are developed in other fields*
Design Research has emerged as an accepted part of educational research, with a strong input from Cognitive Science. Key features include:

- insight focus > products and papers
- realistic classroom situations
- exploring teaching and learning
- theory building

but with

- atypical teachers
- exceptional support
- no claim to wider usability > no direct impact

Engineering research: these products are drafts
Design research > Engineering

For more, see e.g.

Educational Design Research
eds Jan van den Akker, Koen Gravemeier, Susan McKenney, Nienke Nieveen
Routledge 2006

“pragmatic, grounded, interactive, iterative and flexible, integrative, and contextual”

Who does it?
Why isn’t it the mode of development?
“Authors” and publishers

See no need or justification, because

- systematic evaluation is non-existent
- good engineering
  - costs much more
    ~ $20,000 per teaching hour
    still negligible cf system running cost
  - takes time
- powerful tools require more skill

Education ~ “alternative medicine”
Academics?

But the value system favours:

- new ideas *over* reliable research
- new results *over* replication and extension
- trustworthiness *over* generalizability
- small studies *over* major programs
- personal research *over* team research
- first author *over* team member
- disputation *over* consensus building
- papers *over* products and processes
International Society for Design and Development in Education

- www.isddee.org

ISDDE Conference 2009
Cairns, Queensland, Australia
September 27th-30th 2009
Contact:
Kaye Stacey, Conference Chair
k.stacey@edfac.unimelb.edu.au
www.isddee.org

see also

- *Educational Designer*, an e-journal
Teaching materials

- What does your ‘scheme’ cover?

- Who is it designed for? *(realistically!)*

- Moving beyond the published ‘scheme’
  - Selecting replacement units
  - Learning through misconceptions
  - Maintaining some coherence
    - but not too much
Issues for curriculum design

- Is this outward-looking mathematics?
  - few students will become mathematicians
  - math can give them power in their lives
  - does this curriculum do that? for all? (cf ELA)
  - or is it “just math” (RPF)
  - symptoms: all topic focus, no modelling, tasks

- What ‘dimensions of engagement’?
  - many students lack interest in math itself
  - is “make the math interesting” all this does?
  - Does it build ‘mathematical power’
  - symptoms: variety of activities, of tasks (cf ELA)
... and a few more design issues

- Does this develop student autonomy?
  - reliable imitation is not enough to do math
  - what ‘transfer distances’ do the tasks cover?
  - how long are the chains of reasoning?
  - …involving, which problem solving phases?
  - symptoms: no linked phases, similar tasks together

- Does this give teachers enough support?
  - Student-centered teaching is difficult
  - it is easy to overload the teacher
  - what design tactics are used to avoid this?
  - symptoms: teacher in hot seat, centre-stage; no support tactics; too much innovation at once; ……
Professional development pathway

0. Managing the class
   1. Delivering the textbook
   2. Adding good activities (NCTM)
      Many teachers “plateau” here
      For some teachers, this routine expertise then develops into adaptive expertise
      (Hatano, Schoenfeld)

3. Building on where each student is
   Catalyzing and supporting that shift is the core challenge of PD, involves changing:
   1. Knowledge – of math and pedagogy
   2. Orientation – the “classroom contract”
   3. Goals – dimensions of performance
Professional development

- Needs to be materials based, because:
  - Ratio skilled trainers/needy teachers \( \sim 0.001 \)
  - Needed for TTT ‘cascade’ to work

- Design principles:
  - Activity based
  - General principles from specific exemplars (constructive teacher learning)
  - Ongoing, few-year timescale

- Key foci
  - Handling non-routine problems in the classroom
  - Handling discussion non-directively
  - Questioning
  - Changing the “classroom contract”: roles, expectations
Progress so far

- ML can be taught by normal teachers, e.g.
  - Numeracy through Problem Solving (Shell Centre, 1988)
  - Realistic Mathematics Education (FI, 2000)
  - Bowland Maths (UK groups 2008)

- Research-based design pays off:
  - The above examples
  - US evaluation evidence,…..

- Major challenges:
  - Getting the three legs balanced
  - Dynamics of system change