We investigate the factor types of the extremal KMS states for the preferred dynamics on the Toeplitz algebra and the Cuntz-Krieger algebra of a strongly connected finite $k$-graph. For inverse temperatures above 1, all of the extremal KMS states are of type $I_{\infty}$. At inverse temperature 1, there is a dichotomy: if the $k$-graph is a simple $k$-dimensional cycle, we obtain a finite type I factor; otherwise we obtain a type III factor, whose Connes invariant we compute in terms of the spectral radii of the coordinate matrices and the degrees of cycles in the graph.