Recall that a maximal abelian subalgebra $D$ of a von Neumann algebra $M$ is Cartan if the unitaries $U \in M$ satisfying $UDU^* \subseteq D$ span a dense subset of $M$.

Feldman and Moore gave a complete description of Cartan MASAs in von Neumann algebras with separable preduals, in terms of measured equivalence relations. I will present joint work with Allan Donsig and David Pitts in which we describe a bijective correspondence between the family of all Cartan pairs and a certain family of extensions of inverse semigroups.