

**REPORT OF WORKING GROUP 2B**  
**LEARNING IN THE PRESENCE OF TECHNOLOGY**

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This working group of 19 mathematics educators, mathematics education researchers, and mathematicians, critically reflected on what “learning in the presence of technology” looked like in their respective regions and contexts. This working group built upon the work of the Working Group 4 from the previous CMEF held in Montreal in 2003 and whose report is available online at:

[http://www.cms.math.ca/Events/CSMF2003/proceedings/grp\\_4e.pdf](http://www.cms.math.ca/Events/CSMF2003/proceedings/grp_4e.pdf)

As an organizer for the CMEF 2005 Working Group, the following general themes were explored in sequence throughout the working group meetings:

- Exploring the possibilities
- Technology-rich curriculum
- Technology tools and tasks
- Teacher professional learning.

## Exploring the possibilities

The working group aimed at articulating the skills and concepts that they believed could be enhanced through the effective use of various technologies in the learning of mathematics. The four skills that emerged were:

- exploring,
- communicating,
- modeling, and
- reasoning.

A substantial portion of the working group was spent on sharing knowledge and experience on specific uses of the various technological tools that were perceived to contribute to the development of these skills. Some examples are given below.

The use of graphical possibilities, including 3D visualization and dynamic geometry capabilities, has been recognized, for quite some time now, to favour exploration of mathematical objects (possibly very complex) and development of conjectures. What has not always been clear from research is how to articulate the development of such exploration and inductive reasoning with the motivation to proceed to a more formal proof, thereby developing also deductive reasoning. The result of combining a dynamic geometry tool with real-time communication software (in the reported case, Geometer's Sketchpad with Elluminate Live!) was presented as an example of a rich collaborative learning environment that could help increase the motivation for proving, through the possibility for students of sharing constructions and communicating conjectures and ideas, even with a mathematician as remote participant.

As technology allows tackling the complexity of real world situations, many examples were presented by participants to illustrate how specific uses of technology can contribute to the development of modeling.

- The use of video to generate data (e.g. with Logger Pro data collection software) can help develop modeling with graphs (through prediction, validation and interpretation from an experiment) and provide meaning to the mathematical symbolism that would be introduced later on.
- The use of large databases (e.g. from Statistics Canada) combined with data management tools (TinkerPlots, Fathom, or even Excel) can provide an environment to help identify, from graphical exploration, relationships between variables and provide solid ground for the learning of inferential statistics.
- In some introductory calculus courses where a computer algebra system (such as Maple) is used, the focus has moved from developing traditional manipulative skills to modeling and simulating real world situations with functions and differential equations. While requiring from students a structured approach in specifying their problem in a form manageable by the software, this type of technology integration has opened mathematics courses to interdisciplinary problems and the use of a scientific approach, with the possibility of formulating and validating hypotheses.
- The complexity of dynamic systems can be modeled, simulated and analyzed with various computer tools:

- A tool such as Stella allows, through its icon-based interface, a graphical approach to modeling, with the building of diagrams where variables are represented by tanks, and variations (from discrete or continuous processes) by flows.
- Spreadsheets (e.g. Excel) can be used to model and simulate dynamic systems with difference equations.
- Applets are available for use in undergraduate mathematical modeling courses to visualize mathematical representations of objects or processes that are typically used in qualitative analysis of models (phase planes, cobweb diagrams, vector fields, bifurcations, etc.).

It became clear from all these examples that many rich resources currently exist in our Canadian context to support a use of technology that enables mathematical exploration, communication, modeling, and reasoning. However, the consensus of the working group was that although this is necessary, it is not sufficient to ensure the effective use of technology.

Teachers were identified as the key to ensuring that students use technology effectively as an instrument for critical thinking. This idea will be further developed in the final section of this report.

### **Technology-rich curriculum**

Provincially mandated curriculum across Canada is increasingly referencing technology, in each curriculum iteration that is generated. Graphing calculators, predominantly TI 83/84 handheld technology, seem to be the most common tools across Canada in secondary schools and curricula. Apart from applets selected by teachers to illustrate some concepts, other tools used in secondary schools include data management software (Excel, QuatroPro, Fathom, TinkerPlots), and dynamic geometry software (Geometer's SketchPad, Cabri Géomètre). Computer algebra systems (Maple, Derive, TI 89, Voyage 200) are the tools used most frequently in post-secondary mathematics but their use seems to vary dramatically within and between institutions. The working group felt quite strongly that a call needed to go out, particularly to universities, to embed technologies that are connected both to the technologies students used in their secondary mathematics courses/experiences and to the technologies typically used in sectors where mathematics are developed and/or applied. The increasing presence of handheld computers and wireless access could possibly change the landscape in a near future. But the group also agreed that tools alone do not define the adequacy of technology integration in the mathematics curriculum: we must also examine how they are used by teachers and students, and for what content.

With technology, some mathematical content can be covered earlier, but that does not necessarily mean that it should be. We should first assess whether such content is mathematically important, especially with the presence of technology, or if a different content would not be more relevant to developing the skills required in this evolving mathematical practice. Rather than simply looking at how technology can help or accelerate the learning of traditional mathematical content, we must also look at what mathematical problems could now be tackled by students with the use of technology, what concepts and techniques ("new" and "old") would be mobilized in solving these

problems and how the solving of such problems could contribute to the development of a creative, powerful and rigorous mathematical practice.

The idea of “developmentally appropriate” technology was explored. It was felt that more attention needed to be given in our respective curricula to ensure that the technologies of which we encourage or even mandate the use do not put students in a position where they have no way of controlling the results, from either a mathematical or a technological perspective. To help ensure such “developmental appropriateness”, decisions around technology must involve educators who are supposed to use it with their students.

The use of technology in assessment continues to be a complex issue across Canada. On one hand, if technology is never part of the mathematical tasks that are being assessed, then a clear message is being sent that the development of a mathematical practice that makes pertinent use of technology is not a learning goal, recognized by the institution. On the other hand, ensuring fairness and honesty in exams that incorporates technology is constantly challenged by students’ creativity. Issues around access and equity for both classroom and larger scale assessments also remain. All of these elements suggest that technology asks for rethinking assessment in mathematics. One possibility to consider is to provide room for extensive projects, where students are involved in explicit project planning and benefit from checkpoints with the teacher.

The conclusion of the working group was that curriculum and assessment need to evolve with the integration of technology in order to engage students in a mathematical practice that is empowering, meaningful, and coherent, from Kindergarten through postsecondary.

### **Mathematical tasks with technology**

The working group explored the developmental continuum of technology use in the teaching and learning of mathematics, from pre-designed, controlled environments, to open-ended, creative environments. We need to move beyond low-level cognitive and technical tasks to unlock the learning potential of technology for our students and to allow them to take progressive control of the technology they use. Teacher professional learning was identified as the key to ensuring that this move to higher-level uses of technology takes place.

Technology provides mathematics educators with instructional and pedagogical alternatives, which, in turn, provide different ways to get at important mathematical ideas. But we must make sure that students’ autonomy in developing and using these ideas is not hampered by the tools they have used and/or the tasks on which they have worked. As a general rule, whenever possible, we should look for tools and tasks where students can engage progressively in their own constructions. And we should always think of ways to help them look beyond the screen and control the answers they get, by making use of what they know or by creating the need for developing new knowledge. The use of contexts that connect to students’ lives is one way to encourage a critical attitude on the models used and the results obtained.

A complementary approach is to promote the diversity of paths in using the functions of a tool and have students examine, from a mathematical and a technological perspectives, whether they can be considered equivalent. In addition to strengthening the

need for the learning of mathematical properties (as validation criteria), this may require the learning of some of the technical processes (discretization, iteration, etc.) that implement within the tool the mathematical objects and procedures, and can lead to discrepancies with the ideal objects they are supposed to represent.

The learning of these technical processes could, in postsecondary mathematics, lead to the learning of some elements of programming, as true autonomy and creativity in addressing complex problems with some of the most advanced mathematical tools (e.g. Maple or Matlab) still depends upon the ability to program.

### **Teacher professional learning**

It became clear very quickly that teacher professional learning was a critical, complex, long-term issue that needed to be addressed head-on if we were to realize the learning potential of not just technology, but any new tools we wished to introduce into the teaching/learning dynamic of the mathematics classroom. It seems that mathematics educators from across Canada had a collective loathing of the train-the-trainer model, which they had all been subjected to in their recent professional learning experiences. Workshops for secondary teachers around the use of their graphing calculators, organised by the educational technology provider, seem to better address their professional learning needs. Recent interest in professional learning communities has spawned some interesting and innovative approaches to professional learning that may help us define what the necessary conditions are for effective teacher professional learning.

Local, regional, and provincial mathematics conferences traditionally appear to have a significant offering of sessions focused on the technologies that are regionally relevant and available. Whether conferences are, in fact, an effective vehicle for supporting teacher change is an open question. It would seem that some of the conditions that are emerging that are necessary to support teacher professional learning include – time, collaboration, resources, expert support. Schools should become places where, on an on-going basis, in a collaborative fashion, during schooldays and throughout their careers, all teachers can learn to teach mathematics with new technology before students are to learn mathematics with it. This may require early release times for all teachers to learn together. Alternative models to the “whole class going to the lab” should also be envisioned: for example, we should look at ways to use effectively the two or three computers of a classroom, to maintain meaningful and coherent integration of technology in the learning of mathematics.

The working group recommends that a national network of technology experts be created and supported. The working group further recommends that media such as online, streamed video, capturing effective classroom uses of technology be created that can be used to support the professional learning activities of mathematics educators across the country.

## Appendix : Suggested resources for supporting technology integration in mathematics

<b>Graphing Tools</b>	
TI Education	<a href="http://education.ti.com/educationportal/">http://education.ti.com/educationportal/</a>
Peanut Software Homepage	<a href="http://math.exeter.edu/rparris/default.html">http://math.exeter.edu/rparris/default.html</a>
WIMS	<a href="http://wims.unice.fr/wims/wims.cgi?lang=fr">http://wims.unice.fr/wims/wims.cgi?lang=fr</a>
<b>Dynamic Geometry Environments</b>	
Cabri-géomètre	<a href="http://www.cabri.com/v2/pages/fr/index.php">http://www.cabri.com/v2/pages/fr/index.php</a>
AbraCadaBRI	<a href="http://www-cabri.imag.fr/abracadabri/">http://www-cabri.imag.fr/abracadabri/</a>
GS Resource Center	<a href="http://www.keypress.com/sketchpad/">http://www.keypress.com/sketchpad/</a>
King, J. and Schattschneider, D. (Eds.), <i>Geometry Turned On: Dynamic Software in Learning, Teaching and Research</i> , MAA Notes #41, The Mathematical Association of America, 1997.	
<b>Statistics and data</b>	
Statistics Canada Statistique Canada	<a href="http://www.statcan.ca/english/edu/index.htm">http://www.statcan.ca/english/edu/index.htm</a> <a href="http://www.statcan.ca/francais/edu/">http://www.statcan.ca/francais/edu/</a>
Environnement Canada Environment Canada	<a href="http://www.ec.gc.ca/environment_f.html">http://www.ec.gc.ca/environment_f.html</a> <a href="http://www.ec.gc.ca/environment_e.html">http://www.ec.gc.ca/environment_e.html</a>
<b>Spreadsheets</b>	
Spreadsheets and math	<a href="http://sunsite.univie.ac.at/Spreadsite/">http://sunsite.univie.ac.at/Spreadsite/</a>
Excel et le cours de math	<a href="http://www.brunette.brucity.be/iph/math/">http://www.brunette.brucity.be/iph/math/</a>
<b>Computer algebra systems</b>	
Calculatrices TI à l'ÉTS	<a href="http://www.seg.etsmtl.ca/ti/home.html">http://www.seg.etsmtl.ca/ti/home.html</a>
TI Education	<a href="http://education.ti.com/educationportal/">http://education.ti.com/educationportal/</a>
Le saut quantique, Logiciels	<a href="http://www.apsq.org/sautquantique/doss/d-logiciels.html">http://www.apsq.org/sautquantique/doss/d-logiciels.html</a>
Le Stang, Apprendre Maple	<a href="http://alamanya.free.fr/index.html">http://alamanya.free.fr/index.html</a>
Maple Application Center	<a href="http://www.maplesoft.com/applications/index.aspx">http://www.maplesoft.com/applications/index.aspx</a>
Maple Student Center	<a href="http://www.maplesoft.com/academic/students/index.aspx">http://www.maplesoft.com/academic/students/index.aspx</a>
Maple at MIT	<a href="http://web.mit.edu/afs/athena.mit.edu/software/maple/www/home.html">http://web.mit.edu/afs/athena.mit.edu/software/maple/www/home.html</a>
Guin, D. et Trouche, L. (dir.) <i>Calculatrices symboliques : transformer un outil en un instrument du travail mathématique : un problème didactique</i> , Grenoble : La Pensée Sauvage, 2002.	
<b>Other software</b>	
Illuminate	<a href="http://www.illuminate.com/">http://www.illuminate.com/</a>
Vernier Software (LoggerPro)	<a href="http://www.vernier.com/soft/index.html">http://www.vernier.com/soft/index.html</a>
ISEE Systems (Stella)	<a href="http://www.iseesystems.com/index.aspx">http://www.iseesystems.com/index.aspx</a>
<b>General resources</b>	
MathTools	<a href="http://mathforum.org/mathtools/">http://mathforum.org/mathtools/</a>
Key Curriculum Press	<a href="http://www.keypress.com/">http://www.keypress.com/</a>
Masalski, W.J. (Ed.), <i>Technology-Supported Mathematics Learning Environments</i> , NCTM Sixtyseventh Yearbook, 2005.	
Galbraith et al. (Eds.) <i>Mathematical Modelling – Teaching and Assessment in a Technology-Rich World</i> , Horwood Series in Mathematics and Applications, 1998.	