PROBLEMS FOR OCTOBER 2009

Please send your solutions to

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individually as you solve the problems. Electronic files can be sent to *barbeau@math.utoronto.ca*. However, please do not send scanned files; they use a lot of computer space, are often indistinct and can be difficult to download.

It is important that your complete mailing address and your email address appear legibly on the front page. If you do not write your family name last, please underline it.

640. Suppose that $n \geq 2$ and that, for $1 \leq i \leq n$, we have that $x_i \geq -2$ and all the x_i are nonzero with the same sign. Prove that

$$(1 + x_1)(1 + x_2) \cdots (1 + x_n) > 1 + x_1 + x_2 + \cdots + x_n \quad ,$$

641. Observe that $x^2 + 5x + 6 = (x + 2)(x + 3)$ while $x^2 + 5x - 6 = (x + 6)(x - 1)$. Determine infinitely many coprime pairs (m, n) of positive integers for which both $x^2 + mx + n$ and $x^2 + mx - n$ can be factored as a product of linear polynomials with integer coefficients.

642. In a convex polyhedron, each vertex is the endpoint of exactly three edges and each face is a concyclic polygon. Prove that the polyhedron can be inscribed in a sphere.

643. Let n^2 distinct integers be arranged in an $n \times n$ square array ($n \geq 2$). Show that it is possible to select n numbers, one from each row and column, such that if the number selected from any row is greater than another number in this row, then this latter number is less than the number selected from its column.

644. Given a point P, a line $\mathfrak{L}$ and a circle $\mathfrak{C}$, construct with straightedge and compasses an equilateral triangle PQR with one vertex at P, another vertex Q on $\mathfrak{L}$ and the third vertex R on $\mathfrak{C}$.

645. Let $n \geq 3$ be a positive integer. Are there n positive integers $a_1, a_2, \cdots, a_n$ not all the same such that for each i with $3 \leq i \leq n$ we have

$$a_i + S_i = (a_i, S_i) + [a_i, S_i] \quad .$$

where $S_i = a_1 + a_2 + \cdots + a_n$, and where $(\cdot, \cdot)$ and $[\cdot, \cdot]$ represent the greatest common divisor and least common multiple respectively?

646. Let ABC be a triangle with incentre I. Let AI meet BC at L, and let X be the contact point of the incircle with the line BC. If D is the reflection of L in X on line BC , we construct B' and C' as the reflections of D with respect to the lines BI and CI, respectively. Show that the quadrailateral $BCC'B'$ is cyclic.