

PROBLEMS FOR AUGUST 2009

Please send your solutions to

E.J. Barbeau
Department of Mathematics
University of Toronto
40 St. George Street
Toronto, ON M5S 2E4

individually as you solve the problems. Electronic files can be sent to barbeau@math.utoronto.ca. However, please do not send scanned files; they use a lot of computer space, are often indistinct and can be difficult to download.

It is important that your complete mailing address and your email address appear legibly on the front page. If you do not write your family name last, please underline it.

633. Let ABC be a triangle with $BC = 2 \cdot AC - 2 \cdot AB$ and D be a point on the side BC . Prove that $\angle ABD = 2\angle ADB$ if and only if $BD = 3CD$.

634. Solve the following system for real values of x and y :

$$2^{x^2+y} + 2^{x+y^2} = 8$$

$$\sqrt{x} + \sqrt{y} = 2 .$$

635. Two unequal spheres in contact have a common tangent cone. The three surfaces divide space into various parts, only one of which is bounded by all three surfaces; it is “ring-shaped”. Being given the radii r and R of the spheres with $r < R$, find the volume of the “ring-shaped” region in terms of r and R .

636. Let ABC be a triangle. Select points D, E, F outside of $\triangle ABC$ such that $\triangle DBC, \triangle EAC, \triangle FAB$ are all isosceles with the equal sides meeting at these outside points and with $\angle D = \angle E = \angle F$. Prove that the lines AD, BE and CF all intersect in a common point.

637. Let n be a positive integer. Determine how many real numbers x with $1 \leq x < n$ satisfy

$$x^3 - \lfloor x^3 \rfloor = (x - \lfloor x \rfloor)^3 .$$

638. Let x and y be real numbers. Prove that

$$\max(0, -x) + \max(1, x, y) = \max(0, x - \max(1, y)) + \max(1, y, 1 - x, y - x)$$

where $\max(a, b)$ is the larger of the two numbers a and b .

639. (a) Let $ABCDE$ be a convex pentagon such that $AB = BC$ and $\angle BCD = \angle EAB = 90^\circ$. Let X be a point inside the pentagon such that AX is perpendicular to BE and CX is perpendicular to BD . Show that BX is perpendicular to DE .

(b) Let N be a regular nonagon, *i.e.*, a regular polygon with nine edges, having O as the centre of its circumcircle, and let PQ and QR be adjacent edges of N . The midpoint of PQ is A and the midpoint of the radius perpendicular to QR is B . Determine the angle between AO and AB .