Please send your solution to

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no later than February 29, 2008. It is important that your complete mailing address and your email address appear on the front page. If you do not write your family name last, please underline it.

Notes. \( \lfloor x \rfloor \), known as the “floor of \( x \), is the largest integer \( n \) that does not exceed \( x \), i.e., that integer \( n \) for which \( n \leq x < n+1 \). The notation \([PQR]\) denotes the area of the triangle \( PQR \). A geometric progression is a sequence for which the ratio of two successive terms is always the same; its \( n \)th term has the general form \( ar^{n-1} \).

528. Let the sequence \( \{x_n : n = 0, 1, 2, \ldots\} \) be defined by \( x_0 = a \) and \( x_1 = b \), where \( a \) and \( b \) are real numbers, and by

\[ 7x_n = 5x_{n-1} + 2x_{n-2} \]

for \( n \geq 2 \). Derive a formula for \( x_n \) as a function of \( a \), \( b \) and \( n \).

529. Let \( k \), \( n \) be positive integers. Define \( p_{n,1} = 1 \) for all \( n \) and \( p_{n,k} = 0 \) for \( k \geq n + 1 \). For \( 2 \leq k \leq n \), we define inductively

\[ p_{n,k} = k(p_{n-1,k-1} + p_{n-1,k}) \]

Prove, by mathematical induction, that

\[ p_{n,k} = k\sum_{r=0}^{k-1} (-1)^r (k-r)^n \]

530. Let \( \{x_1, x_2, x_3, \ldots, x_n, \ldots\} \) be a sequence is distinct positive real numbers. Prove that this sequence is a geometric progression if and only if

\[ \frac{x_1}{x_2} \sum_{k=1}^{n-1} x_k^{2} = \frac{x_n^{2} - x_1^{2}}{x_2^{2} - x_1^{2}} \]

for all \( n \geq 2 \).

531. Show that the remainder of the polynomial

\[ p(x) = x^{2007} + 2x^{2006} + 3x^{2005} + 4x^{2004} + \cdots + 2005x^{3} + 2006x^{2} + 2007x + 2008 \]

is the same upon division by \( x(x + 1) \) as upon division by \( x(x + 1)^2 \).

532. The angle bisectors \( BD \) and \( CE \) of triangle \( ABC \) meet \( AC \) and \( AB \) at \( D \) and \( E \) respectively and meet at \( I \). If \([ABD] = [ACE]\), prove that \( AI \perp ED \). is the converse true?

533. Prove that the number

\[ 1 + \lfloor (5 + \sqrt{17})^{2008} \rfloor \]

is divisible by \( 2^{2008} \).

534. Let \( \{x_n : n = 1, 2, \cdots\} \) be a sequence of distinct positive integers, with \( x_1 = a \). Suppose that

\[ 2 \sum_{k=1}^{n} \sqrt{x_k} = (n+1)\sqrt{x_n} \]

for \( n \geq 2 \). Determine \( \sum_{k=1}^{n} x_k \).