PROBLEMS FOR SEPTEMBER

Please send your solution to

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no later than October 15, 2007. It is important that your complete mailing address and your email address appear on the front page. If you do not write your family name last, please underline it.

514. Prove that there do not exist polynomials $f(x)$ and $g(x)$ with complex coefficients for which

$$\log_b x = \frac{f(x)}{g(x)}$$

where b is any base exceeding 1.

515. Let n be a fixed positive integer exceeding 1. To any choice of n real numbers x_i satisfying $0 \leq x_i \leq 1$, we can associate the sum

$$\sum \{|x_i - x_j| : 1 \leq i < j \leq n\} .$$

What is the maximum possible value of this sum and for which values of the x_i is it assumed?

516. Let $n \geq 1$. Is it true that, for any $2n+1$ positive real numbers $x_1, x_2, \cdots, x_{2n+1}$, we have that

$$\frac{x_1 x_2}{x_3} + \frac{x_2 x_3}{x_4} + \cdots + \frac{x_{2n+1} x_1}{x_2} \geq x_1 + x_2 + \cdots + x_{2n+1} ,$$

with equality if and only if all the x_i are equal?

517. A man bought four items in a *Seven-Eleven* store. The clerk entered the four prices into a pocket calculator and *multiplied* to get a result of 7.11 dollars. When the customer objected to this procedure, the clerk realized that he should have added and redid the calculation. To his surprise, he again got the answer 7.11. What did the four items cost?

518. Let I be the incentre of triangle ABC, and let AI, BI, CI, produced, intersect the circumcircle of triangle ABC at the respective points D, E, F. Prove that $EF \perp AD$.

519. Let AB be a diameter of a circle and X any point other than A and B on the circumference of the circle. Let t_A, t_B and t_X be the tangents to the circle at the respective points A, B and X. Suppose that AX meets t_B at Z and BX meets t_A at Y. Show that the three lines YZ, t_X and AB are either concurrent (i.e. passing through a common point) or parallel.

520. The *diameter* of a plane figure is the largest distance between any pair of points in the figure. Given an equilateral triangle of side 1, show how, by a stright cut, one can get two pieces that can be rearranged to form a figure with minimum diameter

(a) if the resulting figure is convex (*i.e.* the line segment joining any two of its points must lie inside the figure);

(b) if the resulting figure is not necessaarily convex, but it is connected (*i.e.* any two points in the figure can be connected by a curve lying inside the figure).