The Canadian Mathematical Society La Société mathématique du Canada

The Canadian Mathematical Society

in collaboration with

The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

presents the

Sun Life Financial Canadian Open Mathematics Challenge

Sun Life Financial

Wednesday, November 21, 2007

Time: $2\frac{1}{2}$ hours

Calculators are NOT permitted.

Do not open this booklet until instructed to do so.

There are two parts to this paper.

PART A

This part of the paper consists of 8 questions, each worth 5 marks. You can earn full value for each question by entering the correct answer(s) in the space provided. If your answer is incorrect, any work that you do will be considered for part marks, **provided that it is done in the space allocated** to that question in your answer booklet.

PART B

This part of the paper consists of 4 questions, each worth 10 marks. Finished solutions must be written in the appropriate location in the answer booklet. Rough work should be done separately. If you require extra pages for your finished solutions, paper will be provided by your supervising teacher. Any extra papers should be placed inside your answer booklet. Be sure to write your name and school name on any inserted pages.

Marks are awarded for completeness, clarity, and style of presentation. A correct solution poorly presented will not earn full marks.

NOTES:

At the completion of the contest, insert the information sheet inside the answer booklet.

The names of top scoring competitors will be published on the Web sites of the CMS and CEMC.

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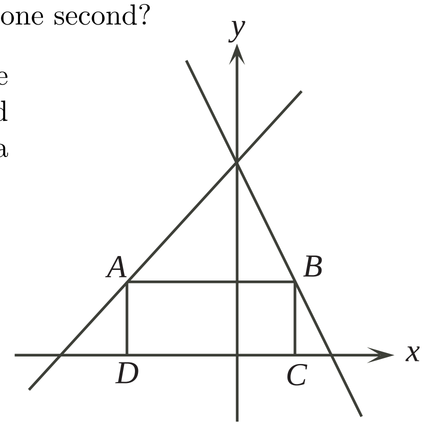
NOTE: 1.		Please read the instructions on the front cover of this booklet.
2.		Write solutions in the answer booklet provided.
3.		It is expected that all calculations and answers will be expressed as exact numbers such as 4π, $2+\sqrt{7}$, etc., rather than as $12.566\ldots$ or $4.646\ldots$.
4.		Calculators are **not** allowed.

PART A

1. If $a = 15$ and $b = -9$, what is the value of $a^2 + 2ab + b^2$?

2. A circular wind power generator turns at a rate of 30 complete revolutions per minute. Through how many degrees does it turn in one second?

3. In the diagram, $ABCD$ is a rectangle with A on the line $y = x + 10$, B on the line $y = -2x + 10$, and C and D on the x-axis. If $AD = 4$, what is the area of rectangle $ABCD$?



4. In June, the ratio of boys to girls in a school was $3:2$. In September, there were 80 fewer boys and 20 fewer girls in the school and the ratio of boys to girls was $7:5$. What was the total number of students at the school in June?

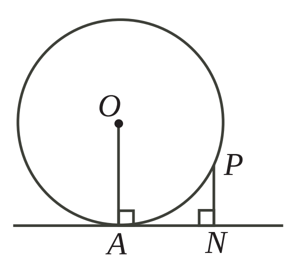
5. The numbers $1, 2, 3, \ldots, 9$ are placed in a square array. The sum of the three rows, the sum of the three columns, and the sum of the two diagonals are added together to form a "grand sum", S.
For example, if the numbers are placed as shown, the grand sum is

1	2	3
4	5	6
7	8	9

$$\begin{aligned} S &= \text{row sums} + \text{column sums} + \text{diagonal sums} \\ &= 45 + 45 + 30 \\ &= 120\ . \end{aligned}$$

What is the maximum possible value of the grand sum S?

6. In the diagram, O is the centre of the circle, AN is tangent to the circle at A, P lies on the circle, and PN is perpendicular to AN. If $AN = 15$ and $PN = 9$, determine the radius of the circle.

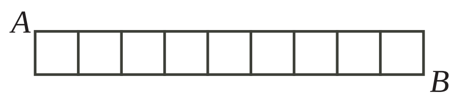


7. Determine all ordered triples of real numbers, (x, y, z), that satisfy the system of equations

$$\begin{aligned} xy &= z^2 \\ x+y+z &= 7 \\ x^2+y^2+z^2 &= 133 \quad . \end{aligned}$$

8. In the diagram, there are 28 line segments of length 1 arranged as shown to form 9 squares. There are various routes from A to B travelling along the segments so that no segment is travelled more than once. Of these possible routes, determine

- the length of route that occurs the most often, and
- the number of different routes of this length.



PART B

1. An arithmetic sequence $a,\ a+d,\ a+2d,\ \ldots$ is a sequence in which successive terms have a common difference d. For example, $2, 5, 8, \ldots$ is an arithmetic sequence with common difference $d = 3$ because $5 - 2 = 8 - 5 = 3$.

   (a) If $x-1$, $2x+2$ and $7x+1$ are the first three terms of an arithmetic sequence, determine the value of x.

   (b) For the value of x from (a), what is the middle term of the arithmetic sequence $x-1,\ 2x+2,\ 7x+1,\ \ldots\ ,\ 72$?

   A geometric sequence $a, ar, ar^2, \ldots$ is a sequence in which successive terms have a common ratio r. For example, the sequence $2, 10, 50, \ldots$ is a geometric sequence with common ratio $r = 5$ because $\dfrac{10}{2} = \dfrac{50}{10} = 5$.

   (c) If $y-1$, $2y+2$ and $7y+1$ are the first three terms of a geometric sequence, determine all possible values of y.

   (d) For each of the values of y from (c), determine the 6th term of the geometric sequence $y-1,\ 2y+2,\ 7y+1,\ \ldots$.

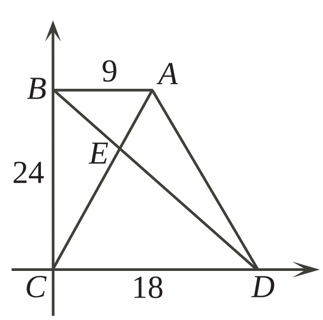
2. In the diagram, $\angle ABC = \angle BCD = 90^\circ$. Also, $AB = 9$, $BC = 24$ and $CD = 18$. The diagonals AC and BD of quadrilateral $ABCD$ meet at E.

   (a) Determine the area of the quadrilateral $ABCD$.

   (b) Show that the ratio $DE : EB = 2 : 1$.

   (c) Determine the area of triangle DEC.

   (d) Determine the area of triangle DAE.



3. Alphonse and Beryl are back! They are playing a two person game with the following rules:

   - Initially there is a pile of N stones, with $N \geq 2$.
   - The players alternate turns, with Alphonse going first. On his first turn, Alphonse must remove at least 1 and at most $N-1$ stones from the pile.
   - If a player removes k stones on their turn, then the other player must remove at least 1 and at most $2k-1$ stones on their next turn.
   - The player who removes the last stone wins the game.

   (a) Determine who should win the game when $N = 7$, and explain the winning strategy.

   (b) Determine who should win the game when $N = 8$, and explain the winning strategy.

   (c) Determine all values of N for which Beryl has a winning strategy. Explain this strategy.

4. A cat is located at C, 60 metres directly west of a mouse located at M. The mouse is trying to escape by running at 7 m/s in a direction 30° east of north. The cat, an expert in geometry, runs at 13 m/s in a suitable straight line path that will intercept the mouse as quickly as possible.

   (a) If t is the length of time, in seconds, that it takes the cat to catch the mouse, determine the value of t.

   (b) Suppose that the mouse instead chooses a different direction to try to escape. Show that no matter which direction it runs, all points of interception lie on a circle.

   (c) Suppose that the mouse is intercepted after running a distance of d_1 metres in a particular direction. If the mouse would have been intercepted after it had run a distance of d_2 metres in the opposite direction, show that $d_1 + d_2 \geq 14\sqrt{30}$.