The Canadian Mathematical Society

in collaboration with

The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

The
Canadian Open Mathematics Challenge

Wednesday, November 26, 2003

Time: $2\frac{1}{2}$ hours

Calculators are NOT permitted.

Do not open this booklet until instructed to do so.
There are two parts to the paper.

PART A
This part of the paper consists of 8 questions, each worth 5. You can earn full value for each question by entering the correct answer in the space provided. Any work you do in obtaining an answer will be considered for part marks if you do not have the correct answer, **provided that it is done in the space allocated** to that question in your answer booklet.

PART B
This part of the paper consists of 4 questions, each worth 10. Finished solutions must be written in the appropriate location in the answer booklet. Rough work should be done separately. If you require extra pages for your finished solutions, paper will be provided by your supervising teacher. Any extra papers should be placed inside your answer booklet.
Marks are awarded for completeness, clarity, and style of presentation. A correct solution poorly presented will not earn full marks.

NOTE: At the completion of the contest, insert the information sheet inside the answer booklet.

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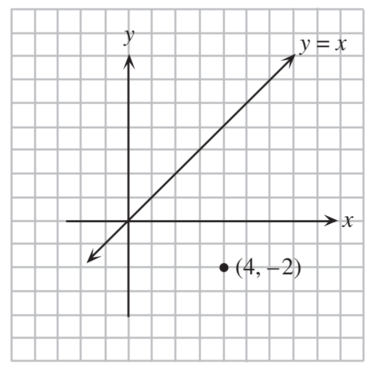
NOTE: 1. Please read the instructions on the front cover of this booklet.
2. Write solutions in the answer booklet provided.
3. It is expected that all calculations and answers will be expressed as exact numbers such as $4\pi, 2+\sqrt{7}$, etc.
4. Calculators are **not** allowed.

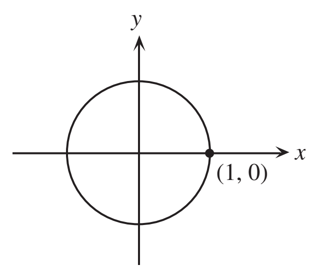
PART A

1. Jeff, Gareth and Ina all share the same birthday. Gareth is one year older than Jeff, and Ina is two years older than Gareth. This year the sum of their ages is 118. How old is Gareth?

2. The point $(4,-2)$ is reflected in the x-axis. The resulting point is then reflected in the line with equation $y = x$. What are the coordinates of the final point?

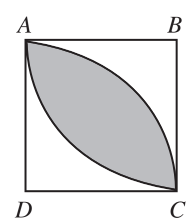


3. A circle of radius 1 is centred at the origin. Two particles start moving at the same time from the point $(1,0)$ and move around the circle in opposite directions. One of the particles moves counterclockwise with constant speed v and the other moves clockwise with constant speed $3v$. After leaving $(1,0)$, the two particles meet first at point P, and continue until they meet next at point Q. Determine the coordinates of the point Q.



4. Two *different* numbers are chosen at random from the set $\{0, 1, 2, 3, 4\}$. What is the probability that their sum is greater than their product?

5. In the diagram, square $ABCD$ has a side length of 6. Circular arcs of radius 6 are drawn with centres B and D. What is the area of the shaded region?



6. The symbol $\lfloor a \rfloor$ means the greatest integer less than or equal to a. For example, $\lfloor 5.7 \rfloor = 5$, $\lfloor 4 \rfloor = 4$ and $\lfloor -4.2 \rfloor = -5$.

Determine all values of x for which $\left\lfloor \frac{3}{x} \right\rfloor + \left\lfloor \frac{4}{x} \right\rfloor = 5$.

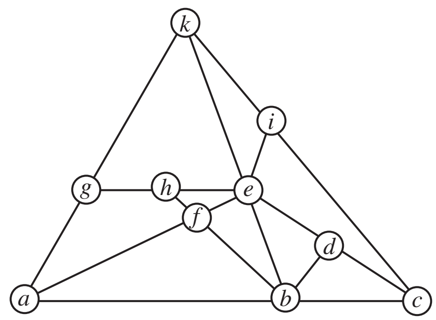
7. Each of the points $P(4,1)$, $Q(7,-8)$ and $R(10,1)$ is the midpoint of a radius of the circle C. Determine the length of the radius of circle C.

8. Determine the number of triples (k, l, m) of positive integers such that

$$k+l+m=97$$
$$\frac{4k}{5}+\frac{5l}{6}+\frac{6m}{7}=82$$

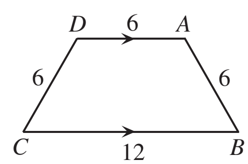
PART B

1. In the diagram shown, whole numbers are to be placed in the ten circles so that the sum of the numbers in the circles along any of the ten straight lines is 15. For example, $a+g+k=15$ and $e+i=15$.



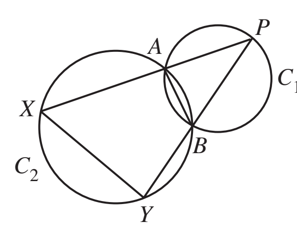
   (a) If $k=2$ and $e=5$, fill in the whole numbers that go in all of the circles in the diagram.
   (b) Suppose that $k=2$ and the value of e is unknown.
      (i) Find a formula for each of b and c in terms of e. A clearly labelled diagram is sufficient explanation.
      (ii) Show that e must be equal to 5.
   (c) Suppose now that $k=x$, where x is unknown. Prove that e must still be equal to 5.

2. A barn has a foundation in the shape of a trapezoid, with three sides of length 6 m, and one side of length 12 m, as shown.



   (a) Determine each of the interior angles in the trapezoid.
   (b) Chuck the Llama is attached by a chain to a point on the outside wall of the barn. Chuck is smarter than the average llama, and so realizes that he can always reach the area between the barn and where the chain is fully extended.
      (i) If Chuck is attached at the point A with a chain of length 8 m, what is the area outside the barn that Chuck can reach?
      (ii) If Chuck is attached at some point P along the wall between A and B with a chain of length 15 m, determine the location of P which restricts Chuck to the *minimum* area.

3. (a) In the diagram, the two circles C_1 and C_2 have a common chord AB. Point P is chosen on C_1 so that it is outside C_2. Lines PA and PB are extended to cut C_2 at X and Y, respectively. If $AB=6$, $PA=5$, $PB=7$ and $AX=16$, determine the length of XY.



   (b) Two circles C_3 and C_4 have a common chord GH. Point Q is chosen on C_3 so that it is outside C_4. Lines QG and QH are extended to cut C_4 at V and W, respectively. Show that, no matter where Q is chosen, the length of VW is constant.

4. The polynomial equation $x^3 - 6x^2 + 5x - 1 = 0$ has three real roots *a*, *b* and *c*.

   (a) Determine the value of $a^5 + b^5 + c^5$.

   (b) If $a < b < c$, show that c^{2004} is closer to its nearest integer than c^{2003} is to its nearest integer.