1. Let $S$ be a subset of $\{1, 2, \ldots, 9\}$, such that the sums formed by adding each unordered pair of distinct numbers from $S$ are all different. For example, the subset $\{1, 2, 3, 5\}$ has this property, but $\{1, 2, 3, 4, 5\}$ does not, since the pairs $\{1, 4\}$ and $\{2, 3\}$ have the same sum, namely 5.

What is the maximum number of elements that $S$ can contain?

2. Call a positive integer $n$ **practical** if every positive integer less than or equal to $n$ can be written as the sum of distinct divisors of $n$.

For example, the divisors of 6 are 1, 2, 3, and 6. Since

\[1=1, \quad 2=2, \quad 3=3, \quad 4=1+3, \quad 5=2+3, \quad 6=6,\]

we see that 6 is practical.

Prove that the product of two practical numbers is also practical.

3. Prove that for all positive real numbers $a$, $b$, and $c$,

\[
\frac{a^3}{bc} + \frac{b^3}{ca} + \frac{c^3}{ab} \geq a + b + c,
\]

and determine when equality occurs.

4. Let $\Gamma$ be a circle with radius $r$. Let $A$ and $B$ be distinct points on $\Gamma$ such that $AB < \sqrt{3}r$. Let the circle with centre $B$ and radius $AB$ meet $\Gamma$ again at $C$. Let $P$ be the point inside $\Gamma$ such that triangle $ABP$ is equilateral. Finally, let the line $CP$ meet $\Gamma$ again at $Q$.

Prove that $PQ = r$.

5. Let $N = \{0, 1, 2, \ldots\}$. Determine all functions $f : N \rightarrow N$ such that

\[xf(y) + yf(x) = (x + y)f(x^2 + y^2)\]

for all $x$ and $y$ in $N$. 