Problem 1
Let \( f(x) = \frac{a^x}{x^a} \). Evaluate the sum
\[
f\left( \frac{1}{1996} \right) + f\left( \frac{2}{1996} \right) + f\left( \frac{3}{1996} \right) + \cdots + f\left( \frac{1995}{1996} \right)
\]

Problem 2
Let \( a, b, \) and \( c \) be positive real numbers. Prove that
\[
a^a b^b c^c \geq (abc)^{\frac{a+b+c}{a+b+c}}.
\]

Problem 3
Define a boomerang as a quadrilateral whose opposite sides do not intersect and one of whose internal angles is greater than 180 degrees. (See Figure displayed.) Let \( C \) be a convex polygon having 5 sides. Suppose that the interior region of \( C \) is the union of \( q \) quadrilaterals, none of whose interiors intersect one another. Also suppose that \( b \) of these quadrilaterals are boomerangs. Show that
\[
q \geq b + \frac{4\pi^2 - \pi}{\pi^2}.
\]

Problem 4
Let \( n \) be a fixed positive integer. Show that for only nonnegative integers \( k \), the diophantine equation
\[
x_1^3 + x_2^3 + \cdots + x_n^3 = y^{3k+2}
\]
has infinitely many solutions in positive integers \( x \) and \( y \).

Problem 5
Suppose that \( u \) is a real parameter with \( 0 < u < 1 \). Define
\[
f(x) = \begin{cases} 
0 & \text{if } 0 \leq x \leq u \\
1 - \left( \sqrt{ux} + \sqrt{(1-u)(1-x)} \right)^2 & \text{if } u \leq x \leq 1
\end{cases}
\]
and define the sequence \( \{u_n\} \) recursively as follows:
\[
u_1 = f(1), \text{ and } u_n = f(u_{n-1}) \text{ for all } n > 1.
\]
Show that there exists a positive integer \( k \) for which \( u_k = 0 \).