PROBLEM 1
A competition involving $n \geq 2$ players was held over $k$ days. On each day, the players received scores of 1, 2, 3, ..., $n$ points with no two players receiving the same score. At the end of the $k$ days, it was found that each player had exactly 26 points in total. Determine all pairs $(n, k)$ for which this is possible.

PROBLEM 2
A set of $\frac{1}{2}n(n+1)$ distinct numbers is arranged at random in a triangular array:

\[
\begin{array}{cccc}
\times & & & \\
& \times & & \\
& & \times & \\
& & & \vdots \\
& & & \\
& & & & \times \\
\end{array}
\]

Let $M_k$ be the largest number in the $k$-th row from the top. Find the probability that

\[M_1 < M_2 < M_3 < \cdots < M_n.\]

PROBLEM 3
Let $ABCD$ be a convex quadrilateral inscribed in a circle, and let diagonals $AC$ and $BD$ meet at $X$. The perpendiculars from $X$ meet the sides $AB$, $BC$, $CD$, $DA$ at $A'$, $B'$, $C'$, $D'$ respectively. Prove that

\[|A'B'| + |C'D'| = |A'D'| + |B'C'|.\]

($|A'B'|$ is the length of line segment $A'B'$, etc.)

PROBLEM 4
A particle can travel at speeds up to 2 metres per second along the $x$-axis, and up to 1 metre per second elsewhere in the plane. Provide a labelled sketch of the region which can be reached within one second by the particle starting at the origin.

PROBLEM 5
Suppose that a function $f$ defined on the positive integers satisfies

\[f(1) = 1, \quad f(2) = 2, \quad f(n + 2) = f(n + 2 - f(n + 1)) + f(n + 1 - f(n)) \quad (n \geq 1).\]

(a) Show that

(i) $0 \leq f(n + 1) - f(n) \leq 1$

(ii) if $f(n)$ is odd, then $f(n + 1) = f(n) + 1$.

(b) Determine, with justification, all values of $n$ for which

\[f(n) = 2^{10} + 1.\]