Canadian Mathematical Olympiad
1987

PROBLEM 1
Find all solutions of \( a^2 + b^2 = n! \) for positive integers \( a, b, n \) with \( a \leq b \) and \( n < 14 \).

PROBLEM 2
The number 1987 can be written as a three digit number \( xyz \) in some base \( b \). If \( x + y + z = 1 + 9 + 8 + 7 \), determine all possible values of \( x, y, z, b \).

PROBLEM 3
Suppose \( ABCD \) is a parallelogram and \( E \) is a point between \( B \) and \( C \) on the line \( BC \). If the triangles \( DEC, BED \) and \( BAD \) are isosceles what are the possible values for the angle \( DAB \)?

PROBLEM 4
On a large, flat field \( n \) people are positioned so that for each person the distances to all the other people are different. Each person holds a water pistol and at a given signal fires and hits the person who is closest. When \( n \) is odd show that there is at least one person left dry. Is this always true when \( n \) is even?

PROBLEM 5
For every positive integer \( n \) show that
\[
[\sqrt{n} + \sqrt{n + 1}] = [\sqrt{4n + 1}] = [\sqrt{4n + 2}] = [\sqrt{4n + 3}]
\]
where \([x]\) is the greatest integer less than or equal to \( x \) (for example \([2.3]\) = 2, \([\pi]\) = 3, \([5]\) = 5).