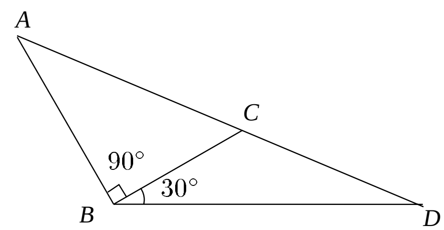
Canadian Mathematical Olympiad
1986

Problem 1

In the diagram line segments AB and CD are of length 1 while angles ABC and CBD are $90°$ and $30°$ respectively. Find AC.



Problem 2

A Mathlon is a competition in which there are M athletic events. Such a competition was held in which only A, B, and C participated. In each event p_1 points were awarded for first place, p_2 for second and p_3 for third, where $p_1 > p_2 > p_3 > 0$ and p_1, p_2, p_3 are integers. The final score for A was 22, for B was 9 and for C was also 9. B won the 100 metres. What is the value of M and who was second in the high jump?

Problem 3

A chord ST of constant length slides around a semicircle with diameter AB. M is the mid-point of ST and P is the foot of the perpendicular from S to AB. Prove that angle SPM is constant for all positions of ST.

Problem 4

For positive integers n and k, define $F(n,k) = \sum_{r=1}^{n} r^{2k-1}$. Prove that $F(n,1)$ divides $F(n,k)$.

Problem 5

Let $u_1, u_2, u_3, \ldots$ be a sequence of integers satisfying the recurrence relation $u_{n+2} = u_{n+1}^2 - u_n$. Suppose $u_1 = 39$ and $u_2 = 45$. Prove that 1986 divides infinitely many terms of the sequence.