Problem 1
Find all positive integers \( w, x, y \) and \( z \) which satisfy \( w! = x! + y! + z! \).

Problem 2
For each real number \( r \) let \( T_r \) be the transformation of the plane that takes the point \((x, y)\) into the point \((2^r x, r 2^r x + 2^r y)\). Let \( F \) be the family of all such transformations i.e. \( F = \{ T_r : r \text{ a real number} \} \). Find all curves \( y = f(x) \) whose graphs remain unchanged by every transformation in \( F \).

Problem 3
The area of a triangle is determined by the lengths of its sides. Is the volume of a tetrahedron determined by the areas of its faces?

Problem 4
Prove that for every prime number \( p \), there are infinitely many positive integers \( n \) such that \( p \) divides \( 2^n - n \).

Problem 5
The geometric mean (G.M.) of a \( k \) positive numbers \( a_1, a_2, \ldots, a_k \) is defined to be the (positive) \( k \)-th root of their product. For example, the G.M. of 3, 4, 18 is 6. Show that the G.M. of a set \( S \) of \( n \) positive numbers is equal to the G.M. of the G.M.’s of all non-empty subsets of \( S \).