Canadian Mathematical Olympiad
1983

Problem 1

Find all positive integers w, x, y and z which satisfy $w! = x! + y! + z!$.

Problem 2

For each real number r let T_r be the transformation of the plane that takes the point (x, y) into the point $(2^r x, r2^r x + 2^r y)$. Let F be the family of all such transformations *i.e.* $F = \{T_r : r \text{ a real number}\}$. Find all curves $y = f(x)$ whose graphs remain unchanged by every transformation in F.

Problem 3

The area of a triangle is determined by the lengths of its sides. Is the volume of a tetrahedron determined by the areas of its faces?

Problem 4

Prove that for every prime number p, there are infinitely many positive integers n such that p divides $2^n - n$.

Problem 5

The geometric mean (G.M.) of a k positive numbers $a_1, a_2, \ldots, a_k$ is defined to be the (positive) k-th root of their product. For example, the G.M. of 3, 4, 18 is 6. Show that the G.M. of a set S of n positive numbers is equal to the G.M. of the G.M.'s of all non-empty subsets of S.