2004 Samuel Beatty Contestant Report: Jacob Tsimerman

45th International Mathematical Olympiad, Athens, Greece

After being asked to write this report, I spent quite a while wondering about what to include in it. Should I just write about Greece and the people I met, discarding any and all math details? That would seem like not including the main part of the IMO, which is after all the competition. On the other hand, just writing about the problems and their different solutions in a very technical manner would be very boring, and one could find that information anyways on some internet site like John Schole's www.kalva.demon.co.uk. So I have decided to try and convey what the IMO is from the perspective of a participant, in hopes of enlightening some people about what Olympiads really are to the people who are writing them.

Now then, that being said, some brief history. The IMO is an annual contest which originated in 1959 and includes over 80 countries as participants. Each country selects 6 students to write a 2 day math contest, each day consisting of 3 mathematical questions and 4.5 hours in which to solve them. This year the IMO was being held in Athens, Greece form July 9-18. The selection process in Canada consists of 3 contests: the APMO, the CMO, and the USAMO. Instead of talking about each of these contests in detail, I will simply say that they are all math contests of a similar structure to the IMO, but yet they are all easier than the IMO. After these contests are written, the results are compared and through some secret process of which neither I nor no student I know is aware, 6 students are chosen as the Canadian IMO team. This year, the 6 chosen students were Oleg Ivrii, Peng Shi, Yufei Zhao, David Rhee, Janos Kramar, and me. I had known Janos and Oleg for a long time, while Yufei, David, and Peng I had not really spent time with in the past.

The camp itself was in Montréal this year, which was nice despite the abundance of people who didn't speak English. Supervising the camp and traveling with us to the IMO were also Chris Small as leader, Ed Wang as deputy leader, and Felix Recio as leader observer. An average day at camp had us solving math problems until about dinner, and than playing chess and dominoes afterwards. The math portion consisted of a short lecture about a specific type of problem, and then several Olympiad problems to work on. Overall it was a very entertaining camp. We got to solve some very elegant problems, and we could always ask Dr.Small for one of his unusual and difficult problems as he seemed to have an endless list of them. It was nice that despite the fact that we spent a lot of time doing math, we also managed to do some hiking and watch a movie as well.

After two weeks, we all flew to Greece for the competition. It was well organized this time, since they gave us 3 days to lose jet lag before the competition. It was fun meeting all the teams, but everyone was still very nervous about the competition. The day before the Olympiad we had the opening ceremony. I thought that the ceremony was overall really well organized, despite repeated references to the contribution of Greece to the field of mathematics and to the similarity between the IMO and the Olympic games. After the ceremony was over, we waited in line to get our dinner, and went to sleep. I can't speak for the others, but sleeping was not something that came to me easily that night. The next day, we bussed to the competition room, sat down at our desks, and at 9:10 the contest began. Now, here's the thing. You've been training for 3 years, solving thousands and thousands of problems, building up experience and getting confidence, learning new methods and techniques; and all you're faced with is a sheet of paper with 3 problems neatly typed on it, and 4.5 hours. This is, to me, the main experience of the IMO. The feeling you get when you realize that all you have to do is solve 3 problems on a sheet of paper, and that nothing else matters. I will now try to summarize the problems, so a reader who is not really interested can skip over the next 2 paragraphs.

Problem 1 was a more or less standard geometry question in which the main idea was proving that 4 points were cyclic. This is fairly routine so this question was overall solved very well. Problem 2 was a very strange polynomial equation, because what you were given was a relation for the polynomial to satisfy with 3 variables a,b,c, but the 3 variables you had to work with had to satisfy a strange constraint: ab+bc+ac=0. Now, a constraint like this is what a contestant detests the most. The reason is not that it makes the question very difficult, but because it limits the ability to play with the relation, and it gives a sense of being stuck. This question had, nevertheless, several ways of approaching it, and was also fairly well done. The main idea was to get an exponential equation and show that the polynomial's degree could not exceed 4. After this, it became a

matter of algebraic manipulation which is easy or hard depending on how much one is used to doing it. The 3rd problem turned out to be the hardest one in the contest. It was a tiling problem, asking for all rectangles tillable with a given shape. This is one of those questions to which you know that there is a relatively simple solution, but you also know that it's probably very hard to stumble upon. The first key step in the problem was to realize that in the tiling the two shapes have to be grouped in pairs. After this, the problem quickly reduced to the interesting case of a rectangle with both sides even but not divisible by 4. At this point, there were two main ways to proceed. The first is very standard, and this was essentially to look for a coloring. This is indeed very standard, but there were very few colorings that solved the problem, and getting the right one was far from easy. The second way of approaching the problem involved breaking symmetry and looking at the tiling from top to bottom, and eventually proving that the number of shapes must be even, which quickly leads to a contradiction. This problem was on the whole done very poorly, with only 9 complete solutions. Seeing as how the solution itself is no more than a few paragraphs, this is really quite astonishing from a sideline perspective. However, it often turns out the hardest questions are the ones which have very short solutions, but which are very non-standard and therefore difficult to think of.

Let's move on to the next day. Problem 4 was an inequality problem which was more or less standard. The main idea was to organize the terms well and apply well known inequalities. A contestant used to dealing with inequalities is used to all this, so this question was also very well solved on the whole. Problem 5, however, was a much more complicated geometry question. Despite the fact that it was found to be extremely difficult, it had several solutions. There were a trigonometry solution, 2 geometry solutions, a solution involving complex numbers, and several others that I heard about. A geometry questions is always a frightening thing, because one can rarely tell whether progress has been made or not. Usually the first sign of something useful is not far from the solution to the problem. It is often the case that one stares at the problem for 4 hours, and ends up getting 0 or 1 points for it. Problem 6 was a number theory question, which essentially had one solution. The question involved the decimal notation of a number, so it was hard to work algebraically. The question asked for all numbers n which had a multiple satisfying a given condition regarding its decimal expansion. This question is also difficult because one first has to guess the solution before trying to prove it. The trick to this question was to try and find a relatively easy multiple that satisfies the condition for most numbers n. The problem is that one does not know how much of the question is just searching for an example, how much is proving existence, and how much is proving non-existence, so one frequently gets mislead and wastes a lot of time. Overall, the contest was very fun to do because every question involved some kind of neat idea, and there is no greater feeling than getting an insight on an Olympiad which makes everything fall into place. I would just like to make one more comment about the problems, and Olympiad problems in general. Discussing problems after one knows several solutions, and trying to solve them under a time restriction never having seen them before are fundamentally different things. Any idea seems simple once you see it on paper, and every question seems easy once the solution is known.

Well, after each day, everyone would start discussing the problems and their solutions; wondering whether mistakes were made and what medal cutoffs are going to be. We all spent a reasonable time looking at the scoreboards, since we were al interested to see how we did. I should also mention the hard work that Dr. Small, Dr. Recio and Dr. Wang put into the coordination sessions, and how grateful we all are to them. In the end, the Canadian team did really well, and none of us went home empty-handed. We all had a good time afterwards on the excursions to Greek sites and talking to all the other teams. We all enjoyed ourselves, despite the raging heat. The closing ceremonies were also well organized, aside form the fact that the medals kept ripping. On the whole, it was a very enjoyable experience, and we all made a lot of friends with people from other teams. I feel obligated to also mention the incredible food that was prepared for us by Greece, and the wondrous dessert that was so kindly supplied by our guide.

It is time to thank some people. I would primarily like to thank Dr.Small, Dr.Recio and Dr.Wang for all of their hard work. Co-ordination sessions aside, dealing with 6 kids for almost a month is quite a task. I would also like to thank Professor Dufour for all his help during our stay in Montreal, and Angela Boukourakis for being a great guide during our stay in Athens. Finally, I would like to thank the CMS, as well as all of its sponsors for making Canada's participation in the IMO possible. The IMO is an unforgettable experience, and I hope that Canadians continue to participate in it and make Canada proud for many years to come.