THE OLYMPIAD CORNER

No. 184

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We begin this number with the problems of the Mock Test for the International Mathematical Olympiad team of Hong Kong. My thanks go to Richard Nowakowski, Canadian Team Leader, for collecting them at the 35^{th} IMO in Hong Kong.

INTERNATIONAL MATHEMATICAL OLYMPIAD 1994

Hong Kong Committee — Mock Test, Part I

Time: 4.5 hours

1. In $\triangle ABC$, we have $\angle C = 2\angle B$. P is a point in the interior of $\triangle ABC$ satisfying $AP = AC$ and $PB = PC$. Show that AP trisects the angle $\angle A$.

2. In a table-tennis tournament of 10 contestants, any two contestants meet only once. We say that there is a winning triangle if the following situation occurs: i^{th} contestant defeated j^{th} contestant, j^{th} contestant defeated k^{th} contestant, and k^{th} contestant defeated i^{th} contestant. Let W_i and L_i be respectively the number of games won and lost by the i^{th} contestant. Suppose $L_i + W_j \geq 8$ whenever the i^{th} contestant beats the j^{th} contestant. Prove that there are exactly 40 winning triangles in this tournament.

3. Find all the non-negative integers x, y, and z satisfying that

$$7^x + 1 = 3^y + 5^z.$$

Mock Test, Part II

Time: 4.5 hours

1. Suppose that $yz + zx + xy = 1$ and x, y, and $z \geq 0$. Prove that

$$x(1-y^2)(1-z^2) + y(1-z^2)(1-x^2) + z(1-x^2)(1-y^2) \leq \frac{4\sqrt{3}}{9}.$$

2. A function $f(n)$, defined on the natural numbers, satisfies:

$$f(n) = n - 12 \text{ if } n > 2000, \quad \text{and} \quad f(n) = f(f(n+16)) \quad \text{if} \quad n \leq 2000.$$

(a) Find $f(n)$.

(b) Find all solutions to $f(n) = n$.

3. Let m and n be positive integers where m has d digits in base ten and $d \leq n$. Find the sum of all the digits (in base ten) of the product $(10^n - 1)m$.

As a second Olympiad set we give the problems of the Final Round of the 45^{th} Mathematical Olympiad written in April, 1994. My thanks go to Marcin E. Kuczma, Warszawa, Poland; and Richard Nowakowski, Canadian Team leader to the 35^{th} IMO in Hong Kong, for collecting them.

45th MATHEMATICAL OLYMPIAD IN POLAND

Problems of the Final Round — April 10–11, 1994

First Day — Time: 5 hours

1. Determine all triples of positive rational numbers (x, y, z) such that $x + y + z$, $x^{-1} + y^{-1} + z^{-1}$ and xyz are integers.

2. In the plane there are given two parallel lines k and l, and a circle disjoint from k. From a point A on k draw the two tangents to the given circle; they cut l at points B and C. Let m be the line through A and the midpoint of BC. Show that all the resultant lines m (corresponding to various points A on k) have a point in common.

3. Let $c \geq 1$ be a fixed integer. To each subset A of the set $\{1, 2, \ldots, n\}$ we assign a number $w(A)$ from the set $\{1, 2, \ldots, c\}$ in such a way that

$$w(A \cap B) = \min(w(A), w(B)) \quad \text{for} \quad A, B \subset \{1, 2, \ldots, n\}.$$

Suppose there are $a(n)$ such assignments. Compute $\lim_{n\to\infty} \sqrt[n]{a(n)}$.

Second Day — Time: 5 hours

4. We have three bowls at our disposal, of capacities m litres, n litres and $m + n$ litres, respectively; m and n are mutually coprime natural numbers. The two smaller bowls are empty, the largest bowl is filled with water. Let k be any integer with $1 \leq k \leq m + n - 1$. Show that by pouring water (from any one of those bowls into any other one, repeatedly, in an unrestricted manner) we are able to measure out exactly k litres in the third bowl.

5. Let $A_1, A_2, \ldots, A_8$ be the vertices of a parallelepiped and let O be its centre. Show that

$$4(OA_1^2 + OA_2^2 + \cdots + OA_8^2) \leq (OA_1 + OA_2 + \cdots + OA_8)^2.$$

6. Suppose that n distinct real numbers $x_1, x_2, \ldots, x_n$ $(n \geq 4)$ satisfy the conditions $x_1 + x_2 + \cdots + x_n = 0$ and $x_1^2 + x_2^2 + \cdots + x_n^2 = 1$. Prove that one can choose four distinct numbers a, b, c, d from among the x_i's in such a way that

$$a + b + c + nabc \leq x_1^3 + x_2^3 + \cdots + x_n^3 \leq a + b + d + nabd.$$

We now give three solutions to problems given in the March 1996 *Corner* as the Telecom 1993 Australian Mathematical Olympiad [1996: 58].

TELECOM 1993 AUSTRALIAN MATHEMATICAL OLYMPIAD
Paper 1

Tuesday, 9th February, 1993

(Time: 4 hours)

6. In the acute-angled triangle ABC, let D, E, F be the feet of altitudes through A, B, C, respectively, and H the orthocentre. Prove that

$$\frac{AH}{AD} + \frac{BH}{BE} + \frac{CH}{CF} = 2.$$

Solution by Mansur Boase, student, St. Paul's School, London, England.

$$\begin{aligned}
\frac{AH}{AD} + \frac{BH}{BE} + \frac{CH}{CF} &= 3 - \left(\frac{HD}{AD} + \frac{HE}{BE} + \frac{HF}{CF}\right) \\
&= 3 - \left(\frac{[BHC]}{[ABC]} + \frac{[CHA]}{[ABC]} + \frac{[AHB]}{[ABC]}\right) \\
&= 3 - \frac{[ABC]}{[ABC]} = 2.
\end{aligned}$$

7. Let n be a positive integer, $a_1, a_2, \ldots, a_n$ positive real numbers and $s = a_1 + a_2 + \cdots + a_n$. Prove that

$$\sum_{i=1}^{n} \frac{a_i}{s - a_i} \geq \frac{n}{n-1} \qquad \text{and} \qquad \sum_{i=1}^{n} \frac{s - a_i}{a_i} \geq n(n-1).$$

Solutions by Mansur Boase, student, St. Paul's School, London, England and Edward T.H. Wang, Wilfrid Laurier University, Waterloo, Ontario. We give the solution by Boase.

$$\sum_{i=1}^{n} \frac{s-a_i}{a_i} = \sum_{i=1}^{n} \left(\frac{s}{a_i} - 1\right) = \sum_{i=1}^{n} \frac{s}{a_i} - n$$

$$\sum_{i=1}^{n} \frac{a_i}{s} = 1 \quad \text{and} \quad \sum_{i=1}^{n} \frac{s}{a_i} \sum_{i=1}^{n} \frac{a_i}{s} \geq (\sum 1)^2 = n^2,$$

by the Cauchy–Schwarz inequality.

Thus

$$\sum_{i=1}^{n} \frac{s}{a_i} \geq n^2.$$

Hence $\displaystyle\sum_{i=1}^{n} \frac{s-a_i}{a_i} \geq n^2 - n = n(n-1)$.

To prove the first inequality, first note that

$$\sum_{i=1}^{n} 1 \sum_{i=1}^{n} a_i^2 \geq \left(\sum_{i=1}^{n} a_i\right)^2 = s^2.$$

Hence $\displaystyle\sum_{i=1}^{n} a_i^2 \geq \frac{s^2}{n}$.

By the Cauchy–Schwarz inequality,

$$\sum_{i=1}^{n} a_i(s-a_i) \sum_{i=1}^{n} \frac{a_i}{s-a_i} \geq \left(\sum_{i=1}^{n} a_i\right)^2 = s^2.$$

Therefore

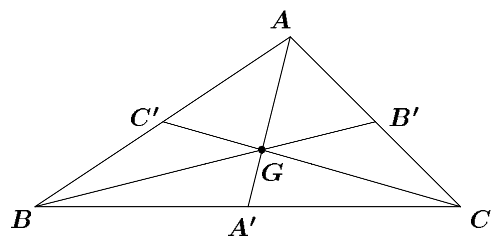
$$\begin{aligned} \sum_{i=1}^{n} \frac{a_i}{s-a_i} &\geq \frac{s^2}{\sum_{i=1}^{n} a_i(s-a_i)} = \frac{s^2}{s\sum_{i=1}^{n} a_i - \sum_{i=1}^{n} a_2^2} \\ &\geq \frac{s^2}{s^2 - \frac{s^2}{n}} = \frac{1}{1-\frac{1}{n}} = \frac{n}{n-1}. \end{aligned}$$

So, both inequalities are proved.

8. [1996: 58] *Telecom 1993 Australian Mathematical Olympiad.*

The vertices of triangle ABC in the xy–plane have integer coordinates, and its sides do not contain any other points having integer coordinates. The interior of ABC contains only one point, G, that has integer coordinates. Prove that G is the centroid of ABC.

Solution by Mansur Boase, student, St. Paul's School, London, England.



By Pick's Theorem

$$\begin{aligned}[ABC] &= 1+3\left(\frac{1}{2}\right)-1 = \frac{3}{2},\\ [ABG] &= 0+3\left(\frac{1}{2}\right)-1 = \frac{1}{2},\\ [BCG] &= \frac{1}{2} \quad \text{and}\\ [CAG] &= \frac{1}{2}.\end{aligned}$$

Therefore

$$\frac{[ABG]}{[ABC]} = \frac{[BCG]}{[ABC]} = \frac{[CAG]}{[ABC]} = \frac{1}{3}.$$

Hence

$$\frac{GA'}{AA'} = \frac{GB'}{BB'} = \frac{GC'}{CC'} = \frac{1}{3}.$$

The unique point satisfying this above is well-known to be the centroid.

Next we give one solution from the Japan Mathematical Olympiad 1993 given in the March 1996 Corner.

2. [1996: 58] *Japan Mathematical Olympiad 1993.*

Let $d(n)$ be the largest odd number which divides a given number n. Suppose that $D(n)$ and $T(n)$ are defined by

$$D(n) = d(1) + d(2) + \cdots + d(n),$$

$$T(n) = 1 + 2 + \cdots + n.$$

Prove that there exist infinitely many positive numbers n such that $3D(n) = 2T(n)$.

Solutions by Mansur Boase, student, St. Paul's School, London, England, and Zun Shan and Edward T.H. Wang, Wilfrid Laurier University, Waterloo, Ontario. We give the solution by Boase.

$$T(n) = \frac{n(n+1)}{2}.$$

Thus we need to prove that there are infinitely many n for which

$$D(n) = \frac{n(n+1)}{3} \qquad \text{so that} \qquad 3D(n) = 2T(n) \qquad \text{holds.}$$

Consider

$$\begin{aligned} D(2^n) &= d(1) + d(3) + \cdots + d(2^n - 1) + d(2) + d(4) + \cdots + d(2^n) \\ &= 1 + 3 + \cdots + (2^n - 1) + d(1) + d(2) + \cdots + d(2^{n-1}) \\ &= 1 + 3 + \cdots + (2^n - 1) + D(2^{n-1}). \end{aligned}$$

Now

$$\begin{aligned} 1 + 3 + \cdots + (2^n - 1) &= \frac{2^n(2^n+1)}{2} - 2\frac{2^{n-1}(2^{n-1}+1)}{2} \\ &= 2^{n-1}(2^n - 2^{n-1}) \\ &= 2^{2n-2}. \end{aligned}$$

Thus $D(2^n) = D(2^{n-1}) + 2^{2n-2}$.

Now, $D(2^1) = 2$ and we shall prove by induction that $D(2^n) = \dfrac{2^{2n}+2}{3}$ for $n \geq 0$.

This holds for $n = 0$ and for $n = 1$. Suppose it holds for $n = k$.

Thus $D(2^k) = \dfrac{2^{2k}+2}{3}$.

Then

$$\begin{aligned} D(2^{k+1}) = D(2^k) + 2^{2k} &= \frac{2^{2k}+2}{3} + 2^{2k} \\ &= \frac{4(2^{2k})+2}{3} \\ &= \frac{2^{2k+2}+2}{3} \end{aligned}$$

and the result follows by induction. Now consider $D(2^n - 2)$.

$$\begin{aligned} D(2^n - 2) &= D(2^n) - d(2^n - 1) - d(2^n) \\ &= \frac{2^{2n}+2}{3} - (2^n - 1) - 1 \\ &= \frac{2^{2n}+2}{3} - 2^n \\ &= \frac{2^{2n} - 3(2^n) + 2}{3} = \frac{(2^n-1)(2^n-2)}{3}. \end{aligned}$$

Thus $D(x) = \dfrac{x(x+1)}{3}$ for $x = 2^n - 2$, and there are infinitely many such x.

Next we turn to comments and solutions from the readers to problems from the April 1996 number of the Corner where we gave the selection test for the Romanian Team to the 34th IMO as well as three contests for the Romanian IMO Team [1996: 107–109].

SELECTION TESTS FOR THE ROMANIAN TEAM, 34th IMO. Part II — First Contest for IMO Team

1st June, 1993

1. Find the greatest real number a such that

$$\frac{x}{\sqrt{y^2+z^2}} + \frac{y}{\sqrt{z^2+x^2}} + \frac{z}{\sqrt{x^2+y^2}} > a$$

is true for all positive real numbers x, y, z.

Solutions by Zun Shan and Edward T.H. Wang, Wilfrid Laurier University, Waterloo, Ontario.

We claim that $a = 2$. Let

$$f(x,y,z) \;=\; \frac{x}{\sqrt{y^2+z^2}} + \frac{y}{\sqrt{z^2+x^2}} + \frac{z}{\sqrt{x^2+y^2}}.$$

We show that $f(x,y,z) > 2$. Since $f(x,y,z) \to 2$ as $x \to y$ and $z \to 0$, the lower bound 2 is sharp. Without loss of generality, assume that $x \geq y \geq z$. Since by the arithmetic–harmonic–mean inequality, we have

$$\frac{\sqrt{z^2+x^2}}{\sqrt{y^2+z^2}} + \frac{\sqrt{y^2+z^2}}{\sqrt{z^2+x^2}} \;\geq\; 2,$$

it suffices to show that

$$f(x,y,z) \;>\; \frac{\sqrt{z^2+x^2}}{\sqrt{y^2+z^2}} + \frac{\sqrt{y^2+z^2}}{\sqrt{z^2+x^2}}$$

or equivalently,

$$\frac{z}{\sqrt{x^2+y^2}} \;>\; \frac{\sqrt{z^2+x^2}-x}{\sqrt{y^2+z^2}} + \frac{\sqrt{y^2+z^2}-y}{\sqrt{z^2+x^2}}.$$

By simple algebra, this is easily seen to be equivalent to

$$\frac{z}{\sqrt{y^2+z^2}(\sqrt{z^2+x^2}+x)} + \frac{z}{\sqrt{z^2+x^2}(\sqrt{y^2+z^2}+y)} \;<\; \frac{1}{\sqrt{x^2+y^2}}. \qquad (1)$$

Since $\sqrt{y^2+z^2} \geq \sqrt{2z^2} = \sqrt{2}\,z$, $\sqrt{z^2+x^2} > x$ and $\sqrt{2}\,x \geq \sqrt{x^2+y^2}$, we have

$$\frac{z}{\sqrt{y^2+z^2}(\sqrt{z^2+x^2}+x)} < \frac{z}{\sqrt{2}\,z(x+x)} = \frac{1}{2\sqrt{2}\,x} \leq \frac{1}{2\sqrt{x^2+y^2}}.$$

Thus to establish (1), it remains to show that

$$\frac{z}{\sqrt{z^2+x^2}(\sqrt{y^2+z^2}+y)} < \frac{1}{2\sqrt{x^2+y^2}}$$

or equivalently

$$\frac{2z}{\sqrt{y^2+z^2}+y} < \sqrt{\frac{z^2+x^2}{x^2+y^2}}.$$

Since

$$\frac{z^2+x^2}{x^2+y^2} = 1 - \frac{y^2-z^2}{x^2+y^2},$$

which is an non-decreasing function of x, we have

$$\frac{z^2+x^2}{x^2+y^2} \geq \frac{z^2+y^2}{2y^2},$$

and thus it suffices to show that

$$\frac{\sqrt{z^2+y^2}}{\sqrt{2}\,y} > \frac{2z}{\sqrt{y^2+z^2}+y},$$

or equivalently

$$y^2+z^2+y\sqrt{y^2+z^2} > 2\sqrt{2}\,yz. \qquad (2)$$

Since $y^2+z^2 \geq 2yz$, we have

$$\begin{aligned} y^2+z^2+y\sqrt{y^2+z^2} &\geq 2yz + y\sqrt{2z^2} \\ &= (2+\sqrt{2})yz > 2\sqrt{2}\,yz \end{aligned}$$

and thus (2) holds. This completes the proof.

2. Show that if x, y, z are positive integers such that $x^2+y^2+z^2 = 1993$, then $x+y+z$ is not a perfect square.

Solutions by Mansur Boase, student, St. Paul's School, London, England; and by Edward T.H. Wang, Wilfrid Laurier University, Waterloo, Ontario. We give Wang's solution.

We show that the result holds for *nonnegative* integers x, y, and z. Without loss of generality, we may assume that $0 \leq x \leq y \leq z$. Then

$$3z^2 \geq x^2+y^2+z^2 = 1993$$

implies that

$$z^2 \geq 665, \quad z \geq 26.$$

On the other hand $z^2 \leq 1993$ implies that $z \leq 44$ and thus $26 \leq z \leq 44$.

Suppose that $x + y + z = k^2$ for some nonnegative integer k. By the Cauchy–Schwarz Inequality we have

$$k^4 = (x + y + z)^2 \leq (1^2 + 1^2 + 1^2)(x^2 + y^2 + z^2) = 5979$$

and so $k \leq \lfloor \sqrt[4]{5979} \rfloor = 8$. Since $k^2 \geq z \geq 26$, $k \geq 6$. Furthermore, since $x^2 + y^2 + z^2$ is odd, it is easily seen that $x + y + z$ must be odd, which implies that k is odd. Thus $k = 7$ and we have $x + y + z = 49$.

Let $z = 26 + d$, where $0 \leq d \leq 18$. Then

$$\begin{aligned} x + y = 23 - d \Rightarrow y \leq 23 - d \Rightarrow x^2 + y^2 &\leq 2y^2 \\ &\leq 2(23 - d)^2 = 1058 - 92d + 2d^2. \end{aligned} \tag{1}$$

On the other hand, from $x^2 + y^2 + z^2 = 1993$ we get

$$x^2 + y^2 = 1993 - z^2 = 1993 - (26 + d)^2 = 1317 - 52d - d^2. \tag{2}$$

From (1) and (2), we get

$$1317 - 52d - d^2 \leq 1058 - 92d + 2d^2$$

or

$$3d^2 - 40d \geq 259$$

which is clearly impossible since $3d^2 - 40d = d(3d - 40) \leq 18 \times 14 = 252$. This completes the proof.

Remark: It is a well-known (though by no means easy) result in classical number theory that a natural number n is the sum of three squares (of nonnegative integers) if and only if $n \neq 4^l(8k + 7)$ where l and k are nonnegative integers. Since $1993 \equiv 1 \pmod 8$ it can be so expressed and thus the condition given in the problem is not "vacuously" true. In fact, 1993 can be so expressed in more than one way; for example,

$$\begin{aligned} 1993 &= 0^2 + 12^2 + 43^2 \\ &= 2^2 + 15^2 + 42^2 \\ &= 2^2 + 30^2 + 33^2 \\ &= 11^2 + 24^2 + 36^2. \end{aligned}$$

These representations also show that the conclusion of the problem is *false* if we allow x, y, and z to be negative integers; e.g. if $x = -2$, $y = -30$, $z = 33$ then $x^2 + y^2 + z^2 = 1993$ and $x + y + z = 1^2$; and if $x = -11$, $y = 24$, $z = 36$ then $x^2 + y^2 + z^2 = 1993$ and $x + y + z = 49 = 7^2$.

4. Show that for any function $f : \mathcal{P}(\{1, 2, \ldots, n\}) \to \{1, 2, \ldots, n\}$ there exist two subsets, A and B, of the set $\{1, 2, \ldots, n\}$, such that $A \neq B$ and $f(A) = f(B) = \max\{i \mid i \in A \cap B\}$.

Comment by Edward T.H. Wang, Wilfrid Laurier University, Waterloo, Ontario.

The problem, as stated, is clearly incorrect since for $\max\{i : i \in A \cap B\}$ to make sense, we must have $A \cap B \neq \emptyset$. For $n = 1$ clearly there are no subsets A and B with $A \neq B$ and $A \cap B \neq \emptyset$. A counterexample when $n = 2$ is provided by setting $f(\emptyset) = f(\{1\}) = f(\{2\}) = 1$ and $f(\{1,2\}) = 2$. This counterexample stands if $\max$ is changed to $\min$. The conclusion is still incorrect if $A \cap B$ is changed to $A \cup B$. A counterexample would be $f(\emptyset) = 2$ and $f(\{1\}) = f(\{2\}) = f\{(1,2)\} = 1$.

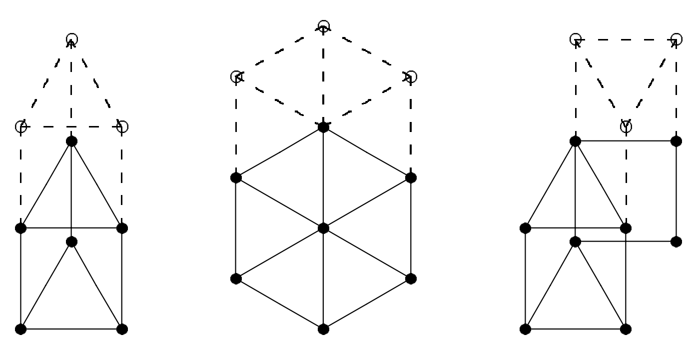
Part III — Second Contest for IMO Team

2nd June, 1993

3. Prove that for all integer numbers n, with $n \geq 6$, there exists an n–point set M in the plane such that every point P in M has at least three other points in M at unit distance to P.

Solution by Zun Shan and Edward T.H. Wang, Wilfrid Laurier University, Waterloo, Ontario.

The three diagrams displayed below illustrate the existence of such a set. M_1 is for all $n = 3k$, M_2 is for all $n = 3k+1$ and M_3 is for all $n = 3k+2$, where $k = 2, 3, 4, \ldots$. In each diagram, the solid lines connecting two points all have unit length and the dotted lines, also of unit length, indicate how to construct an $(n + 3)$–point set with the described property from one with n points.



M_1 $(n = 6, 9, 12, \ldots)$ M_2 $(n = 7, 10, 13, \ldots)$ M_3 $(n = 8, 11, 14, \ldots)$

Part IV — Third Contest for IMO Team

3rd June, 1993

1. The sequence of positive integers $\{x_n\}_{n\geq 1}$ is defined as follows: $x_1 = 1$, the next two terms are the even numbers 2 and 4, the next three terms are the three odd numbers 5, 7, 9, the next four terms are the even numbers 10, 12, 14, 16 and so on. Find a formula for x_n.

Solutions by Mansur Boase, student, St. Paul's School, London, England; and by Zun Shan and Edward T.H. Wang, Wilfrid Laurier University, Waterloo, Ontario. We give the solution of Shan and Wang.

We arrange the terms of the sequence $\{x_n\}$ in a triangular array according to the given rule so that the 1st level consists of a single 1 and for all $k \geq 2$, the k^{th} level consists of the k consecutive even (odd) integers that follow the last odd (even) integer in the $(k-1)^{\text{st}}$ level:

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 2 & & 4 & & & \\ & & 5 & & 7 & & 9 & & \\ & 10 & & 12 & & 14 & & 16 & \\ 17 & & 19 & & 21 & & 23 & & 25 \end{array}$$

For any given $n \in N$, let l_n denote the unique integer such that

$$\binom{l_n}{2} < n \leq \binom{l_n+1}{2};$$

that is,

$$\frac{(l_n-1)l_n}{2} < n \leq \frac{l_n(l_n+1)}{2}. \qquad (1)$$

Note that l_n is simply the number of the level which x_n is on. We claim that

$$x_n = 2n - l_n. \qquad (2)$$

When $n = 1$, clearly $l_1 = 1$ and thus $2n - l_n = 1 = x_1$. Suppose (2) holds for some $n \geq 1$. There are two possible cases:

Case (i): If $n + 1 \leq \binom{l_n+1}{2}$; that is, if x_{n+1} and x_n are on the same level, then $l_{n+1} = l_n$ and hence

$$x_{n+1} = x_n + 2 = 2n - l_n + 2 = 2(n+1) - l_{n+1}.$$

Case (ii): If $n+1 > \binom{l_n+1}{2}$ then x_n is the last number on the l_n^{th} level and x_{n+1} is the first number on the l_{n+1}^{th} level. Thus

$$l_{n+1} = l_n + 1$$

and

$$x_{n+1} = x_n + 1 = 2n - l_n + 1 = 2(n+1) - l_{n+1}.$$

Hence, by induction, (2) is established. From (1) we get

$$l_n^2 - l_n < 2n \leq (l_n+1)^2 - (l_n+1).$$

Solving the inequalities, we easily obtain

$$l_n < \frac{1+\sqrt{1+8n}}{2} \leq l_n + 1.$$

Hence

$$l_n = \left\lceil \frac{1+\sqrt{1+8n}}{2} \right\rceil - 1 \qquad (3)$$

where $\lceil x \rceil$ denotes the least integer greater than or equal to x (that is, the ceiling function). From (2) and (3) we conclude that

$$x_n = 2n + 1 - \left\lceil \frac{1+\sqrt{1+8n}}{2} \right\rceil.$$

That completes this number of the *Corner*. Send me your nice solutions as well as Olympiad contests.