DIMITRIS KOUKOULOPOULOS, Université de Montréal
Groups structures of elliptic curves over finite fields
It is known that an elliptic curve $E$ over a finite field $\mathbb{F}_{p}$ admits a group structure which is abelian and has rank at most 2 . Therefore there are integers $m$ and $k$ such that the group of points of $E$ over $\mathbb{F}_{p}$ is isomorphic to $\mathbb{Z} / m \mathbb{Z} \times \mathbb{Z} / m k \mathbb{Z}$. In the converse direction, Rück characterized which pairs of integers $(m, k)$ can arise this way. It is then natural to ask how many of such pairs exist with $m \leq M$ and $k \leq K$. Call the number of such pairs $S(M, K)$. Banks, Pappalardi and Shparlinski studied the size of $S(M, K)$, which they related to a problem about the existence of primes in short arithmetic progressions. Based on standard heuristics about primes, they made a conjecture about the size of $S(M, K)$ and proved some partial results towards it. In this talk, I will discuss recent progress in this problem which leads to an improvement of the results of Banks, Pappalardi and Shparlinski, as well as to a proof of their conjecture in certain ranges of $M$ and $K$. This is joint work with V . Chandee, C. David and E. Smith.

