BRUCE GILLIGAN, University of Regina, Regina, SK S4S 0A2
Kaehler homogeneous manifolds

Let G be a connected complex Lie group and H a closed, complex subgroup of G. In this talk we will assume that the complex homogeneous manifold $X := G/H$ is Kähler. Kähler homogeneous manifolds X are completely understood if X is compact or the metric is G-invariant. The situation is also understood if the group G is semisimple, solvable, or a direct product $S \times R$ of its radical R with a maximal semisimple subgroup S of G. Attempts to construct examples of noncompact manifolds X homogeneous under a nontrivial semidirect product $G = S \ltimes R$ with a not necessarily G-invariant Kähler metric motivated this work.

In this setting the S-orbit $S/S \cap H$ in X is Kähler. Thus $S \cap H$ is an algebraic subgroup of S. The Kähler assumption on X ought to imply the S-action on the base Y of any homogeneous fibration $X \to Y$ is algebraic too. Natural considerations allow a reduction to the case where $H = \Gamma$ is a discrete subgroup and there is a homogeneous fibration $X = G/\Gamma \to G/I =: Y$ with I° an abelian, normal subgroup of G and the fiber $I^\circ/(I^\circ \cap \Gamma)$ a Cousin group, *i.e.*, a complex Lie group with no nonconstant holomorphic functions. We prove an algebraic condition does hold in the homogeneous manifold $Y = \hat{G}/\hat{\Gamma}$, where $\hat{G} := G/I^\circ$ and $\hat{\Gamma} := I/I^\circ$. Namely, we show that an element $\hat{g} \in \hat{\Gamma}$ of infinite order lying in a semisimple subgroup $\hat{S}$ of $\hat{G}$ is an obstruction to the existence of a Kähler metric on X. So if X is Kähler, then $\hat{S} \cap \hat{\Gamma}$ is finite.

As a consequence, if the group $\hat{G}$ is a linear algebraic group whose radical $\hat{R}$ is a vector group and the representation of $\hat{S}$ on $\hat{R}$ is linear with no nonzero invariant vector, then G/Γ cannot be Kähler. An example of such a group $\hat{G}$ is the affine group of $\mathbb{C}^n$ for $n > 1$.