Report of Working Group 6 – Mathematics and Intuition

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In mathematics, two seemingly opposite mental activities are inextricably linked: an aspect of conscious thought variously known as “reason”, “logic”, “deduction”, etc. and the largely subconscious currents sensed as “inklings”, “insight”, “gut feeling”,”intuition” and the like. If you leave out one or the other, you no longer have mathematics, but fantasy or pedantry, respectively. Obviously the conscious side is more easily described and formalized than the other one – which we have chosen too “intuition”, because it covers (at least) the initial “groping” stage of problem-solving as well as the final “flash of illumination”.

Our meetings were framed by a presentation and discussion of meanings of the word intuition. The term is rooted in Western mystical traditions. Derived from the Latin for “to look within”, an intuition was originally understood as a moment of profound connection to the eternal but obscured truths of the cosmos. Intuitions, that is, were taken as a vital aspect of knowledge.

This situation changed during the Age of Reason, with explicit and focused efforts to cleave ‘knowledge’ from its ancient associations to the mysterious and the unprovable. The two principal movements of the time—rationalism (analytic philosophy) and empiricism (analytic science)—were grounded in the claim that truth had to be either laid bare in the light of reason or rendered observable through replicable demonstrations. In the process, vision-based metaphors associated with reason and measurement (e.g., insight, –clarity, illumination, inspect, suspect, expect) came to be privileged over tactile-based notions associated with intuition (e.g., feeling, sensation, hunch) in references to knowledge.

Modern school mathematics is, in the main, a product of rationalist culture. As such, it has tended to incorporate the privilege of reason and the ignoring -or suppressing of intuition. Such emphases might be seen to underpin the much-lamented loss of meaning in contemporary school mathematics. However, contrary to popular narrative, meaning is not something that was eroded away in mathematics instruction. It was deliberately discarded. When teaching methods and curricula were defined for modern school mathematics, the explicit emphases were on formal expression and mechanical proficiency, not relevance and sensibility.

In scholarly circles, attitudes toward intuition began to change through the 20th century, prompted by the conceptual contributions of phenomenology, psychoanalysis, poststructuralism, ecological discourses, and non-western worldviews. In brief, there has been an emergent realization that most of an individual’s cognitive processes are never present to consciousness. That is, little of what a person (or collective) knows (or experiences) is ever rendered explicit, much lest framed as formal proposition.

In view of this, a pressing question for mathematics education becomes, *How might teachers embrace, nurture and educate intuition?* It was noted in our discussions, however, that this question might be overly optimistic, given that school mathematics continues to be framed by the commitments and obsessions of rationalist thought. Perhaps, it was suggested, a more appropriate question at the moment is, *How do we avoid the suppression or extinction of intuition?* These questions, two sides of the same coin, led us to discuss a related even more fundamental question, *Is intuition vital to sense making?*

Sense making involves such things as, choice, visualization, doubt, anticipation, anxiety, hope, confidence, the authentic use of knowledge, problem solving, problem posing, inquiry, experimentation, investigation, exploration, exercising one's imagination and creativity, indulging one's curiosity, the search for connections and patterns, conjecturing, reflection, assessment, meta-cognition, 'playing with inklings' (by both the teacher and students, interacting). In each of these actions or reactions we saw intuition playing a vital role.

We discussed how teachers attend to and nurture intuition by providing students with opportunities to experience such activities. One shared example involved an adolescent girl who'd been diagnosed as learning disabled in mathematics for most of her schooling career—yet, after a relatively brief period of engaging in rich mathematical activities while ignoring formal symbolic manipulations (that is, of attending to the development of intuitions), she was re-diagnosed as ahead of grade level.

We discussed how teachers can stifle / extinguish / kill intuition by involving students in tasks that neither make sense nor stimulate sense making. Samples of such tasks are readily found in text-books and tests. (Encouragement for the development of such materials can be found in curriculum that are basically many long, grade by grade, lists of finely-specified / over-specified content.) Our discussions here included thinking of over-engineered curriculum, prescriptive texts and rote learning as destroyers of intuition.

We saw intuition as so important that its presence or absence in a classroom can be used as a litmus test of the health of a classroom's sense-making / learning environment. Oriented by this conviction, we briefly discussed possible changes to the culture of school mathematics that might support the development of learners' intuitive powers. Two themes were prominent: current curricula and teacher preparation.

First, and foremost, we agreed that current curricula tend to be over-specified. Because these curricula are often defended in terms of necessary preparation for university mathematics courses, we felt that a possible course of action for the CMS would be to develop and publish some sort of position statement on the sorts of experiences, competencies and dispositions that would be desirable of students entering undergraduate programs with significant mathematics components. Such a statement, we felt (intuited), would be a useful tool in the reshaping of increasingly unwieldy programs of study across Canada.

Our second recommendation, regarding courses in mathematics for teachers, is hinged to the possibility of the first one. If teachers are to teach in ways that support the development of learners' intuitions, it seems reasonable to argue that they must be involved in such learning experiences themselves. Ball & Bass (in these Proceedings) offer some direction on this issue. This topic has also been a prominent theme at annual meetings of the Canadian Mathematics Education Study Group.