

BOOK REVIEWS

The World According to Operator Algebraists

Book review by Ola Bratteli, University of Oslo

Lectures on Operator Theory

B.V. Rajarama Bhat, George A. Elliott
and Peter A. Fillmore (eds)

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xi + 323 pp



The theory of algebras A on a Hilbert space H which are closed under the adjoint operation $a \rightarrow a^*$ has at the outset two different branches with very different flavors, according to whether the algebras are closed in the topology defined by the operator norm $a \rightarrow \|a\|$ or in the topology defined by the strong semi-norms $a \rightarrow \|a\xi\|$ for $\xi \in H$. These two types of algebras are called C^* -algebras and von Neumann algebras, and their study may be labelled “non-commutative topology” and “non-commutative measure theory”, respectively. These names reflect the fact that the study of the two types of algebras are as different as the study of the two topics topology and measure theory. The names also indicate that these algebras may, at least in principle, provide the same fundamental conceptual framework for studying non-commutative or quantum phenomena as topology and measure theory do for classical phenomena.

This description of the field of operator algebras was at least apt when the reviewer entered the subject before 1970, at which time the bible to get into the the subject were Dixmier’s two books on the two branches of the subject. At this time it was still more or less possible for an ordinary mortal to get an overview of the whole subject. Since then it has grown and scattered into various sub-specialities, and cross-fertilized with other topics to such an extent that it has become difficult to keep abreast of all developments. Dixmier’s books have reference lists containing all the papers on operator algebras up to the printing date, and as late as 1980 Dick Kadison was still able to collect all the troops in the subject to a three week conference on operator algebras in Kingston

which covered all the specialities up to then. At present it is certainly not possible to arrange a comprehensive conference on the subject, and much less possible for one author (maybe excepting Alain Connes) to write one book encompassing it all.

Instead the editors of the present volume have had the idea of dishing out the subject in small pieces to several authors, so that each of them could write an account of his or her piece of the cake. There are altogether 25 authors. With the exception of the last two sections, the book is written in a uniform encyclopedic style, and it is not even stated which author is responsible for which part of the book. The book is based on lectures given by the authors in the program “Operator Algebras and Applications” which was held at the Fields Institute during the heyday of George Elliott’s classification program for amenable C^* -algebras in 1994-95. The Fields Institute was then based in Waterloo.

The book covers most of the topics in operator algebras:

Part 1, about C^* -algebras, contains basic theory of positivity, K -theory, tensor products, crossed products, free products, dilation theory and operator spaces, applications to quantum statistical mechanics, and applications to Toeplitz operators. One can always look at an encyclopedic book like this from the viewpoint of the devil to find some missing topic to complain about, and I missed some basic material about C^* -dynamical systems in the context of locally compact groups: Arveson spectrum and all the results on conjugacy, outer conjugacy, etc., which developed from Connes’s classification of factors in the seventies. This seems to be material that is falling into oblivion now, even to the extent that old results are re-proved.

Part 2, about von Neumann algebras, contains basic structure, type II_1 factors, a long section on Connes’s and Haagerup’s proof on the equivalence of injectivity and hyperfiniteness, an introduction to sub-factors and the Jones index, the Tomita-Takesaki theory of the modular group, Voiculescu’s theory of free products of von Neumann algebras and Arveson’s and Powers’ theory of semi-groups of endomorphisms of $B(H)$, a surprisingly difficult subject which is still under development. It was maybe wise to avoid a full scale treatment of Connes’ classification of injective factors, especially in view of Takesaki’s much awaited book, which finally seems to be forthcoming now. I missed a chapter about the Italian version of quantum field theory and relation to sub-factors here, but you cannot win them all.

Part 3, is about the classification of C^* -algebras, both for finite and infinite algebras. Eberhard Kirchberg, Chris Phillips and Mikael Rørdam made great strides in the classification of infinite C^* -algebras just during the Waterloo year, and Elliott and his coworkers were prolifically producing

new results on the classification of finite C^* -algebras both before and after this year, to the extent that this subject maybe presently shows signs of elephantiasis and a need for fresh ideas and concepts.

Part 4 contains Irina Stevens' Ph.D. thesis on hereditary sub-algebras of certain simple non real rank zero C^* -algebras, which is a nice illustration of Elliott's classification techniques for inductive limits.

Part 5 is a 90 page richly illustrated article by Adrian Ocneanu on paths on Coxeter diagrams and sub-factors. This article is particularly welcome in view of the paucity of written communication from Ocneanu, and it is based on videotapes of several of his lectures in the period 1995 - 1998.

Overall this is a very nicely and surprisingly uniformly written book which is of interest both for the novice and the expert in operator algebras. It is aimed at giving an overview of the subject, and as mentioned it covers most subjects, although one may always miss things like spectral triples and non-commutative geometry, quantum groups and I don't know what. Maybe the subject of operator algebras presently lacks a really new concept to speed up development (or at least it may look like that to someone old and infirm like the reviewer), like Connes' spectral analysis of automorphisms after 1970, Jones' sub-factors in 1980 and Voiculescu's free algebras around 1990. It may be hoped that the book will inspire some young researcher to new invention.