Abelian groups and representations

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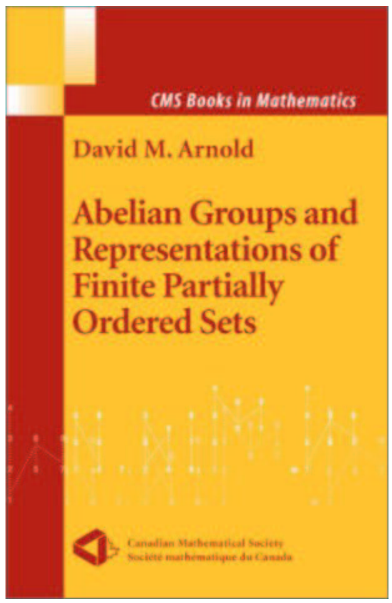
Abelian groups and representations of finite partially ordered sets
by David Arnold
CMS Books in Mathematics/Ouvrages de Mathematiques de la SMC
Springer-Verlag, New York, 2000
xii+244 pp



A recurring theme in classical mathematics is the existence and consequences of inter-relations between different mathematical structures. This is especially true in algebra. This is also the theme of this monograph: an exposition of inter-relations between representations of finite partially ordered sets and abelian groups. Emphasis is placed throughout on classification theorems, descriptions of the objects up to isomorphism, model-theoretic characterizations and computations of representation types, a measure of when classification is feasible. Historically, the idea of group representations occurs naturally in linear algebra e.g. the study of "matrix problems", such as finding canonical forms or showing the equivalence of two matrices. This research monograph by a pioneer in the field of group representations of posets, offers an excellent and much-needed introduction to this active area of research.

The monograph has a total of eight chapters. Topics include: Representations of posets over a field, Torsion-free Abelian groups, Butler groups, Representations over a discrete valuation ring, Almost completely decomposable groups, Representations over fields and exact sequences, Finite rank Butler groups, Applications of representations and Butler groups.

Let k be a field and S a finite poset. A representation of S over k is defined to be a category with objects $U = (U_o, U_i : i \text{ in } S)$, where U_o is a (distinguished) finite dimensional k-vector space, each U_i a subspace of U_o, and if $i \leq j$ in the poset S, then U_i is contained in U_j. Morphisms are k-linear transformations $f : U_o \to U_o$ with $f(U_i)$ a subset of U_i for each i in S. Chapter I is an elementary introduction to fundamental properties of representations of finite posets over a field. Basic notions of countable torsion-free abelian groups and relationships between Butler groups of finite rank and representations of finite posets over a field are discussed in the next two chapters. Recall that a (finite rank) Butler group is a homomorphic image of a finite direct sum of subgroups of the rational numbers $\mathbf{Q}$ under addition. From the model theory point of view, the class of all such groups is the smallest class of torsion-free abelian groups that contains all rank-1 groups and is closed under isomorphism, finite direct sums, pure subgroups and torsion-free homomorphic images. Among the researchers in this area, it is well-known David Arnold was mainly responsible for "popularizing" Butler groups through several papers and his 1981 book on this topic.

Most classical books on Abelian groups were written before the main development of Butler groups and group representations of finite posets. So this is the unique book of this kind. In this book, we witness a wealth of interaction between such general topics as linear algebra, rings and modules, categories, Abelian groups (both torsion and torsion-free), Boolean algebras and even combinatorics. At the end of each chapter, the author provides some exercises, useful notes, chapter summaries and a brief guide to the published literature. With over 210 references, this monograph provides a rich resource of information for a variety of readers from advanced graduate students of algebra to experts working in this field. Some open problems are also mentioned. The book ends with showing clearly how the representation theory of posets can be applied to the study of valuated p-groups, an extremely useful topic for further study of this branch of algebra. In conclusion, I have no hesitation to say that Arnold's book is a valuable contribution to the subject of representations of posets. Advanced graduate students will find it an accessible introduction to the subject, and established algebraists will be able to use the information contained here in their own research.