A Miscellany for Mathematicians and Others
Book Review by David Borwein, University of Western Ontario

Mathematical Conversations – Selections from The Mathematical Intelligencer
compiled by Robin Wilson and Jeremy Gray
Springer-Verlag New York, 2000
vii+ 488 pages

This book is a diverse collection of forty articles which appeared in *The Mathematical Intelligencer* over the past twenty years. They were chosen by Robin Wilson and Jeremy Gray to present to a general mathematical audience and interested others what it is in mathematics that makes it so useful and intriguing. Wilson and Gray express their own views as follows:

“...there is a need for journals that display in an accessible way what mathematics is and what it can do. Popularization wins friends for the subject. It brings in students, it informs colleagues in other fields, and it helps assure politicians and taxpayers that public money is being well and responsibly spent; it can refresh professional mathematicians who necessarily dig deep into their own studies but still wish to know what their neighbours are doing, and it helps locate and affirm the place of mathematics in the collective cultural activity of humanity.”

They view *The Mathematical Intelligencer* to be such a journal, and their selection of articles is their personal choice of materials to fulfill their aim of reaching and interesting a wide audience. This is, of course, an extremely difficult task since the mathematical knowledge of the target group ranges from
almost complete ignorance to specialist expertise. The choice of topics reflects very much the tastes of the selectors. What they consider to be elegant mathematics may appeal to like-minded experts but is usually gibberish to the mathematically “challenged” and may even appear to be dull and tedious to experts in disparate mathematical fields.

Wilson and Gray have divided the book into seven parts and attempted to make the first and last accessible to the non-mathematical. In this they have been only partially successful. For obvious reasons, I’ll not review the material in all seven parts in depth or comprehensively, but will cherry pick to convey some sense of what the book covers.

Part one, entitled Interview and Reminiscences, contains lots of interesting material concerning the views, and in some cases the activities, of the mathematicians Michael Atiyah, Stephen Smale, Jean-Pierre Serre, Julia Robinson, and the physicist C.N. Yang; it also includes anecdotes about Stefan Bergman, Abram Besicovitch, Kurt Gödel, Solomon Lefschetz, and Norbert Wiener. Atiyah is reported to have said “I don’t pay very much attention to the importance of proofs. I think it is more important to understand something.” But then he admits “a proof is important to check your understanding, that’s all. It is the last stage in the operation - an ultimate check - but it isn’t the primary thing at all.” Many mathematicians, myself included, would disagree with this view of the place of proofs in mathematics. Smale describes his outspoken very left-wing political activism at the time of his receipt of the Fields medal in Moscow in 1966. A story is told about Wiener’s absentmindedness: His family moved house and Wiener was given a key to the new house when he left for work on the day of the move. After work he returned to his old house and was dismayed when he found that his key did not open the door and the house appeared empty. He ran up to a child coming down the street and cried “Little girl, I’m very upset. My family has disappeared and my key won’t fit the lock.” She replied, “Yes, daddy. Mommy sent me for you.”

Part seven, entitled History of Mathematics, deals inter alia with the life of Kurt Gödel, Louis de Branges’ proof of the Bieberbach conjecture, the bitter controversy between David Hilbert and L.E.J. Brouwer around the editorship of Mathematische Annalen, and the present state of Hilbert’s 23 problems. Much of this part would certainly be inaccessible to the non-mathematical public.

The other five parts of the book are addressed to the mathematically knowledgeable.

Part two, entitled Algebra and Number Theory, has six articles, including one on Faltings’ proof of the Mordell Conjecture. Another is an essay by M. Ram Murty on Artin’s Conjecture for Primitive Roots in which Murty quotes the following insightful comment by Artin:

“We all believe that mathematics is an art. The author of a book, the lecturer in a classroom tries to convey the structural beauty of mathematics to his readers, to his listeners. In this attempt, he must always fail. Mathematics is logical to be sure, each conclusion is drawn from previously derived statements. Yet the whole of it, the real piece of art, is not linear; worse than that, its perception should be instantaneous. We have all experienced on some rare
occasions the feeling of elation in realizing that we have enabled our listeners to see at a moment’s glance the whole architecture and all its ramifications.”

Number theory of the analytical kind is the subject of R.C. Vaughan’s article, *Adventures in Arithmetick, or: How to Make Good Use of a Fourier Transform*, which connects such seemingly disparate themes as the Riemann Hypothesis and Fermat’s Last Theorem via investigations into Goldbach’s conjecture.

Parts three, four, five, six are entitled *Analysis, Applied Mathematics, Arrangements and Patterns, Geometry and Topology* respectively. Each part contains five or six articles covering a wide range of subjects.

The analysis part deals to quite an extent with counter-intuitive results, such as the Banach-Tarski Theorem which states that it is possible to decompose a sphere $S$ into two spheres $S_1$ and $S_2$, each of which is congruent to $S$, and various constructions of space filling curves due to Peano, Osgood, Sierpiński, Lebesgue and others; there is also a description of Jeff Xia’s proof in 1988 of Painlevé’s long-standing conjecture concerning an aspect of the $n$-body problem.

The applied mathematics part includes articles on string theory, solitons, the relevance of topology to the behaviour of yeast, and one by J. Lambek describing how certain key results in theoretical physics can be expressed concisely in the language of quaternians.

The part on arrangements and patterns deals with both two-dimensional (tessellations) and three-dimensional (crystallography and solid-state physics) patterns, and ends with an article by H. S. M. Coxeter on the work of two sculptors using hollow triangles.

The geometry and topology part contains an article on the topology of 4-manifolds which includes descriptions of the proofs of Michael Friedman and Simon Donaldson which led to the verification of Poincaré’s conjecture concerning smooth 4-manifolds being homeomorphic to a 4-sphere, and gained them Fields Medals in 1986; in addition, there are articles on minimal surfaces, braid and link theory, and hyperbolic 3-manifolds.

To sum up, the book contains a wealth of material with various parts accessible to differing groups of individuals. It is not a book that one would read from cover to cover, but anyone with curiosity and some basic mathematical knowledge could browse through it and pick parts which would be interesting and instructive. I’m afraid, however, that politicians and bureaucrats with little or no mathematical background are unlikely to be driven to funnel more funds into mathematics as a result of trying to read the book. The book would make an ideal gift for anyone mathematically inclined.