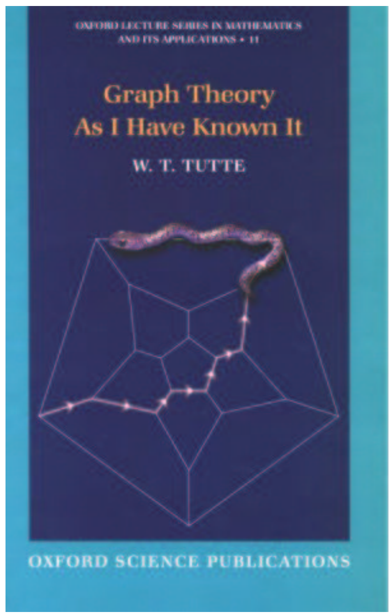
Insights from the Father of Graph Theory

Book Review by Ralph Stanton, University of Manitoba

Graph Theory As I Have Known It
By W.T. Tutte
Clarendon Press, Oxford, 1998
156 pages.



For over sixty years, Bill Tutte has worked in Graph Theory and he can truly be called the father of the subject. His contributions have been many and varied, and have been of fundamental profundity and importance in the development of the subject. This fascinating book is an account of some parts of the theory in which he took special interest, and he reveals how he was led to many of the theorems and proofs for which he is famous.

The way in which the book came to be written is an interesting story in its own right. Professor Tutte gave a set of lectures at the University of Waterloo in 1984, just before his retirement, and these lectures had the same title as this book. U.S.R. Murty had the happy idea of seeing that these lectures appeared in print. Professor Tutte has updated his remarks from the 1984 version, and the book is now available to stimulate the interest of all graph theorists as well as those who have marginal interests in the subject.

I shall try to give the flavour of the book by giving a brief summary of each chapter and then ending up with an over-all view.

Chapter 1, on "Squaring the Square", tells how Bill Tutte, Leonard Brooks, Cedric Smith, and Arthur Stone, inspired by the mathematical recreational books of Rouse Ball and H.E. Dudeney, enthusiastically, in the 1930s, attacked the problem of dissecting a square into smaller but unequal squares. They discovered the connections of this problem with Kirchhoff's famous laws for electrical networks. It is now known that the smallest such square is unique and has order 21. To me, the most important result of this research is that Bill Tutte felt the fascinations of graph theory and left Chemistry, his undergraduate study, to pursue a career in Mathematics.

Chapter 2 of the book is entitled "Knights Errant." Bill explains how the problem of a Knight's tour on a chessboard leads to the more general problem of finding spanning subgraphs, and how the interest of that knight, Sir William Rowan Hamilton, on spanning circuits of the dodecahedral graph led to the theory of hamiltonian circuits.

Chapter 3 is entitled "Graphs within Graphs" and deals with f-factors in graphs. Naturally much of the discussion is devoted to the important case of 1-factors, that is subgraphs in which each vertex has valence unity. Bill explains how he obtained the theorem on the existence of a 1-factor in a graph, and how Maunsell later gave an alternative proof of the result.

Chapter 4, on "Unsymmetrical Electricity", deals with the dissections of equilateral triangles and parallelograms into unequal equilateral triangles. This is the analogue of the work in the first chapter on dissecting a square into smaller unequal squares.

Chapter 5 is called "Algebra in Graph Theory" and deals with many topics, especially the chromatic polynomial, the dichromatic polynomial, and the flow polynomial of a graph.

Chapter 6, on "Symmetry in Graphs," deals with rotors, stators and cages, and introduces that traveller "Serpens," the snake that moves along the edges of a graph under different rules for different problems (the cover of the book actually shows a graph with Serpens making his way through it).

Chapter 7, entitled "Graphs on Spheres," deals with planar graphs and leads to a discussion of Brooks's Theorem, Hadwiger's Conjecture, and the Four-Colour Theorem. It ends with an important sketch of the theory of bridges in a graph.

Chapter 8: "The Cats of Cheshire." This chapter explains how Bill was led to the definition and study of matroids. Personally, I feel that, as well as being the father of graph theory, he deserves to also be called the father of matroid theory.

Chapter 9 explains the problem of "Reconstruction," the reconstruction of some property P of a graph from the known values of P for the subgraphs of the original graph.

Chapter 10, on "Planar Enumeration," describes how Bill solved many of the difficult and important problems in graphical enumeration for which he is so well known. In particular, he explains the importance of "rooting" in graphs, that is, specialization of a particular, vertex, edge, or face.

Chapter 11 is on "The Chromatic Eigenvalues" and includes Bill's notable result that $P(\tau, \tau\sqrt{5})$ is a constant multiple of $P^2(\tau, \tau^2)$, where t is the golden number defined by $\tau^2 = t + 1$. This led him to consider the relation of the Beraha numbers $B_n = 2 + 2cos(2\pi/n)$ to chromatic polynomials.

Chapter 12, entitled "In Conclusion," was written recently. In it, Bill looks back on the results outlined in the earlier chapters, and comments on developments that have taken place since he originally wrote these chapters for his 1984 course.

The bare summary of the chapters that I have just given can not do justice to either the style or the content of the book. Bill Tutte is an entertaining writer, and this work is marked by gentle wit and humour. Bill has an unusual ability to tell a story in a way that lets the reader share the enthusiasm that Bill felt in pursuing his research and that also lets the reader see some of the reasoning that underlay the creative ideas that spurred the developments of the results.

Some 20 years ago, I had the privilege of being a co-editor of the "Selected Papers of W.T. Tutte" (published by the Charles Babbage Research Centre, Box 272, St Norbert Postal Station, Winnipeg, R3V 1L6). At the time, I was struck by the enthusiasm and humour that Bill imparted to the commentaries he contributed concerning the genesis of each of his papers, the development of his ideas, and the relationship of the various results. The present volume is in exactly the same vein as his commentaries on the individual papers in his "Selected Papers"; the narrative provides one with insight into the workings of the mind of one of the most original and creative research mathematicians of our time.

Mathematics, and especially Graph Theory, is very fortunate that Bill Tutte is still with us and still active. His final Chapter in this present volume is marked by the same keen insight that he has displayed for well over 60 years. I am personally very grateful to him for serving as the first President (1990-1996) of the Institute of Combinatorics and its Applications; the growth and success of that organization owes much to his example and his leadership. At the British Combinatorial Conference in Canterbury (July, 1999), Bill delivered the Rado Lecture to a packed auditorium and showed that he had lost none of his ability to entrance the listeners with his own enthusiasm for Graph Theory and its manifold aspects.