Mayhem Problems

To be eligible for this month's MAYHEM TAUNT, solutions must be post-marked *before* **January 1, 2003, and attached to each solution of each problem must be a completed student information sheet.**

M51. Proposed by the Mayhem Staff.

You have a deck with cards numbered 1 through 25. You perform the following operations on the deck:

- you place the top card on the bottom of the deck.
- you place the new top card on the bottom of the deck.
- you flip the new top card face up on the table.

You continue this process until all cards are face up on the table. Find the order of the cards in the deck if, when the process is performed, the cards get laid out on the table in the order 1, 2, 3, $\ldots$, 25.

M52. Proposed by J. Walter Lynch, Athens, GA, USA.

You have two coins. One is a normal half dollar and the other is a fake half dollar with a head on both sides. You randomly toss one of the coins into a drawer and the other coin into another drawer. A man comes into the room and opens one of the drawers. He looks in and sees a head. Question: What is the probability that he is seeing the coin with two heads?

M53. Proposed by the Mayhem Staff.

A circular path is surrounded by 17 stepping stones numbered 0, 1, 2, $\ldots$, 16. Sally starts on stone 0 and moves 1 step to stone 1, then 4 steps to stone 5, then 9 steps to step 14 and continues in the following pattern until at last she moves 2002^2 steps and stops (to rest). What stone is Sally standing on while she rests?

M54. Proposed by Gary Tupper, Pedagoguery Software Inc., Terrace, BC.

An ellipse with major axis AB and foci F and F' is inscribed in a circle with diameter AB and centre C. P is a point on the ellipse and D is a point on the circle so that radius CD bisects FP. Show that line DP is tangent to the ellipse.

Pedagoguery Software has offered a copy of their software GrafEq *to the first correct solution received by the MAYHEM problems editor.*

M55. Proposed by the Mayhem Staff.

Find the sum of the first 2002 terms in the following sequence

$$1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, \ldots.$$

M56. Proposed by Vedula N. Murty, Dover, PA, USA.

Prove the identity

$$\left(\sum \sin A\right)^2 - \left(1 + \sum \cos A\right)^2 \;=\; 4\cos A \cos B \cos C\,,$$

where the sums are cyclic and $A + B + C = \pi$.