Mayhem Problems

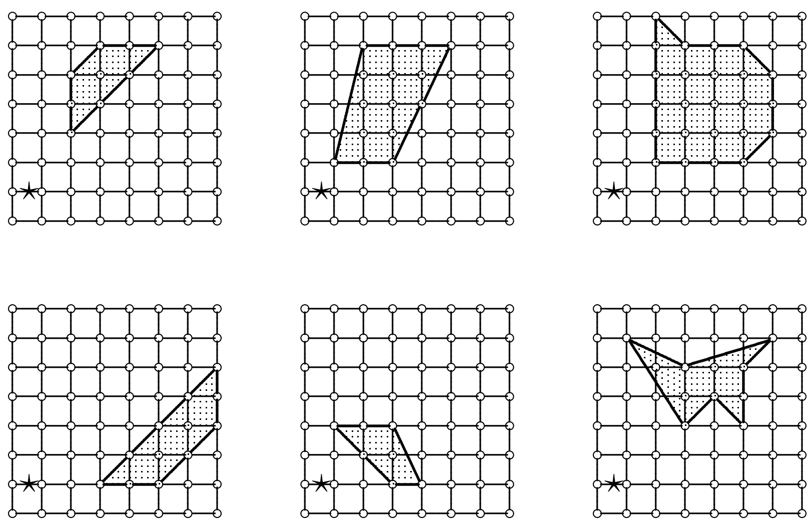
To be eligible for the May 2002 MAYHEM TAUNT, solutions must be postmarked *before* **September 1, 2002, and attached to each solution of each problem must be a completed student information sheet.**

M45. *Proposed by a Canadian Customs of£cer, Pearson International Airport, Toronto, Ontario*.

A 10 metre long ladder is leaning upright against a wall, touching the edge of a cubic box. The box itself is put against the wall and measures 2 cubic metre. What is the height of the top of the ladder from the ground?

M46. *Proposed by Eckard Specht, Otto-von-Guericke-University Magdeburg, Germany*.

The lattice polygons in the upper row of the £gure are characterized by a common property, the lower ones by the reverse. Which property is it?



M47. *Proposed by Bill Sands, University of Calgary, Calgary, Alberta*.

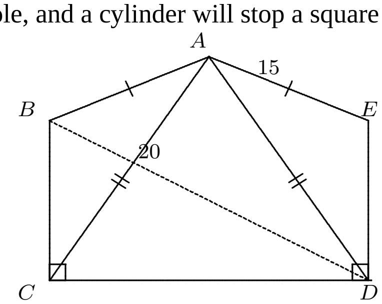
(a) Find all monic quadratic polynomials $x^2 + ax + b$ with integer roots, where 1, a, b is an arithmetic progression.

(b) Prove that there are no real numbers a, b, c such that 1, a, b, c is an arithmetic progression and $x^3 + ax^2 + bx + c$ has all real roots.

M48. *Proposed by J. Walter Lynch, Athens, GA, USA.*

Tell how to make a single stopper that will stop a square hole, a round hole, and a triangular hole, and will pass through each.

If one wanted to give a hint, he might point out that a pyramid will stop a square hole and a triangular hole, and a cylinder will stop a square hole and a round hole.



M49. *Proposed by K.R.S. Sastry, Bangalore, India*.

The £gure shows a Heron pentagon in which the sides, the diagonals and the area are natural numbers.

(a) $AB = AE = 15$, $AC = AD = 20$ and $BCDE$ is a rectangle. Find the length of BD.

(b) Give a set of general expressions for the sides, the diagonals and the area to generate an in£nite family of such Heron pentagons $ABCDE$ as in the £gure.

M50. *Proposed by the Mayhem Staff.*

This question is a bit of a variation of a well known and used problem. There are forms of the question where you want to use four 4's and some operations to make as long a list of values as possible. Thus

$$\frac{4+4}{4+4} \;=\; 1\,, \qquad\quad 4+4-\sqrt{4}-4 \;=\; 2\,,$$

and so on. It is popular to use the digits of the year in such a problem (although, we will have to deal with a couple of zeros for a while).

The problem is to make as many numbers as possible using **up to** £ve π's. Thus some acceptable results would be:

$$\frac{\pi+\pi+\pi}{\pi} \;=\; 3\,, \qquad\quad \left\lfloor \sqrt{\pi^{\pi}} - \pi + \frac{\pi}{\pi} \right\rfloor ! \;=\; 6\,.$$