Non-Linear Control Theory
Théorie de contrôle non-linéaire
(Org: **Andrew Lewis** and/et **Abdol-Reza Mansouri** (Queen's))

MIREILLE BROUCKE, Department of Electrical and Computer Engineering, University of Toronto, Toronto, Ontario, Canada
Geometric Control of Patterned Linear Systems

In this talk we present a class of linear control systems called patterned linear systems, where every matrix in the state space representation of the system shares the same set of eigenvectors.

Equivalently, each system matrix is a polynomial function of a common base matrix. This class includes for example, circulant systems, which are systems comprising a closed chain of identical subsystems that are interconnected in a repeated pattern. Some examples of applications of these systems are mobile networks, paper machines, and the approximation of partial differential equations. The control of circulant systems has been studied by previous researchers using the matrix algebra properties of circulant matrices. Our class is broader than just circulants, and we study patterned systems using abstract algebra, specifically the observation that a set of matrices with common eigenvectors has useful relationships with a set of invariant subspaces. The description of the class in terms of subspaces allows these systems to be studied under geometric control theory. In particular, the objective is to find feedback controllers to solve some of the classic problems of geometric control such as the restricted regulator problem, while preserving the pattern of the system. We conclude with a discussion of several applications of the results.

OVIDIU CALIN, Department of Mathematics, Eastern Michigan University, Ypsilanti, Michigan
Non-Holonomic Systems and Sub-Riemannian Geometry

Sub-Riemannian geometry is the underlying model for non-holonomic systems in a similar way as Riemannian geometry is the framework for classical dynamical systems. For instance, the position of a ship on a sea is determined by three parameters: two position coordinates and an orientation angle. Therefore, the ship's position can be described by a point on the manifold $R^2 \times S^1$. When the ship navigates from position A to position B, it describes a curve on the aforementioned manifold that is tangent to a certain 2-dimensional non-integrable distribution. In general, if this distribution satisfies the Chow's bracket generating condition, any to points A and B can be joined by such a curve. One can ask what is the shortest distance a ship can navigate from one position to another. Because of constraints, the shortest distance is neither a straight line, nor a classical geodesic.

It is a solution of the Euler–Lagrange equations of a certain associated Lagrangian, and they are called sub-Riemannian geodesics. Their length defines the Carnot–Caratheodory metric on the coordinates manifold $R^2 \times S^1$. Similar sub-Riemannian models can be associated with other systems with constraints, such as the rolling penny or rolling ball on a plane, a skater or slide on a plane or a bicycle. The problem of existence of sub-Riemannian geodesics between any two points constitutes one of the research trends in the field. The interested reader can also consult the book *Sub-Riemannian Geometry: General Theory and Examples*, by Calin, et al., published by Cambridge University Press, 2009.

DONG EUI CHANG, Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario, Canada
A Short Proof of the Pontryagin Maximum Principle on Manifolds

I will present a short proof of the Pontryagin Maximum Principle (PMP) on smooth manifolds, using the Whitney embedding theorem, the tubular neighborhood theorem and the PMP in $\mathbb{R}^n$.

VELIMIR JURDJEVIC, Department of Mathematics, University of Toronto, Toronto, Ontario, Canada
Hamiltonian systems on Lie groups and Symmetric spaces

Lie groups G with an involutive automorphism σ provide a natural setting for variational problems that illuminate the theory of integrable Hamiltonian systems. In my talk I will attempt to justify this assertion by focusing on the following problem:

Let $\mathfrak{g}$ denote the Lie algebra of G and let $\mathfrak{g} = \mathfrak{p}+\mathfrak{k}$ denote the Cartan decomposition induced by the Lie algebra automorphism σ_*. Assume the existence of a positive definite quadratic form $\langle\ ,\ \rangle$ on $\mathfrak{k}$ and let A denote a fixed element in Cartan space $\mathfrak{p}$.

Then consider the problem of minimizing $\int_0^1 \frac{1}{2}\langle U(t), U(t)\rangle\, dt$ over the curves $g(t)$ in G that satisfy the given boundary conditions $g(0) = g_0$, $g(T) = g_1$ and are the solutions of following left invariant differential system

$$\frac{dg}{dt}(t) = g(t)\bigl(A + U(t)\bigr)\, dt$$

with $U(t)$ a bounded and measurable curve in $\mathfrak{g}$.

This "optimal control problem" admits well defined solutions for A "regular" and the Maximum Principle of optimal control induces a class of Hamiltonians H on the dual $\mathfrak{g}^*$ of $\mathfrak{g}$. The integrability of the Hamiltonian system defined by H is intimately linked to the "solvability" of the above problem.

It follows, as will be demonstrated, that H is integrable when the quadratic form $\langle\ ,\ \rangle$ is proportional to the Cartan–Killing form. In particular I will show that the most classical integrable systems, such as Jacobi's geodesic problem on the ellipsoid, Kepler's gravitational problem or the elastic problems of Kirchhoff can be seen as particular cases of the above situation.

The talk will end with some open questions.

MANFREDI MAGGIORE, Dept. of ECE, University of Toronto, 10 King's College Road, Toronto, ON M5S 3G4
Reduction Theorems for Stability of Closed Sets in Finite-Dimensional Dynamical Systems, with Application in Control Theory

We investigate the Seibert–Florio reduction problem for finite-dimensional dynamical systems: given two closed positively invariant subsets of the state space, $\Gamma_1 \subset \Gamma_2$, assuming that Γ_1 is either stable, semi-attractive, or semi-asymptotically stable relative to Γ_2, find conditions under which Γ_1 enjoys the same properties relative to the state space. We present reduction theorems which extend, in the finite-dimensional setting, Seibert and Florio's results for compact Γ_1, and illustrate their relevance in Control Theory.

KIRSTEN MORRIS, Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario, Canada
Optimal control of a mechnical system with play

We study the stabilization of a second order differential equation with control occurring through a play operator. This is a model for a mechanical system such as a shaft or gear system with mechanical play on the fittings. Since the differential equation is discontinuous in the state variable the usual existence theory does not apply. It is shown directly that a Caratheodory solution exists for all continuous controls. An optimal control problem of minimizing a cost associated with these dynamics is formulated. It is shown that a viscosity solution to the Hamilton–Jacobi–Belman PDE exists, and hence an optimal control. A scheme for computation of the optimal control is presented along with some numerical results.

This is joint work with Carmeliza Navasca, Clarkson University.