Commutative Algebra and Algebraic Geometry
Algèbre commutative et géométrie algébrique
(Org: **Ragnar-Olaf Buchweitz** (Toronto), **Graham Leuschke** (Syracuse) and/et **Greg Smith** (Queen's))

LUCHEZAR L. AVRAMOV, University of Nebraska
Koszul modules over small graded rings

The title refers to rings defined by more than $\binom{e}{2}$ quadrics in e indeterminates. It will be shown that generic modules with certain types of Hilbert series have linear resolutions.

This is joint work with Srikanth Iyengar and Liana Şega.

SUNIL CHEBOLU, University of Western Ontario
Which finite p-groups are like a finite product of fields?

A ghost in a tensor triangulated category is a map that induces the zero map on all homotopy groups. We identify the derived categories of commutative rings and the stable module categories of finite p-groups that do not have non-trivial ghosts. In particular, we will solve the riddle posed in the title: which finite p-groups are like a finite product of fields?

This is joint work with Dan Christensen and Jan Minac.

KIA DALILI, Dalhousie University, Chase Building, Halifax, NS, B3H 3J5
The reconstruction conjecture for edge ideals

Given a simple graph G on n vertices, let the deck of G be the collection of unlabeled subgraphs of G obtained by removing one vertex from G. An invariant of a graph is called reconstructible if it has the same value for any two graphs with the same deck. Graph theorists have studied reconstruction of combinatorial invariant of G as an strategy to prove the isomorphism class of G is reconstructible. We prove that it is possible to reconstruct several algebraic properties of the edge ideal from the deck of G. These properties include Krull dimension, Hilbert function, and all graded Betti numbers $\beta_{i,j}$ where $j < n$.

HARM DERKSEN, University of Michigan, Ann Arbor
A symmetric function generalization of the Tutte polynomial

I will discuss ideals associated to subspace arrangements. The Hilbert series of the product of the ideal corresponding to the subspaces is a combinatorial invariant. Using this, I will construct a generalization of the Tutte polynomial.

NEIL EPSTEIN, University of Michigan, Ann Arbor, MI
Pieces of closures

I will discuss recent work on breaking closure operations into specific, useful parts in various different ways. This applies to integral closure, tight closure, and other closures, and includes "special (parts of)" closures and "interiors" of closures. This is useful, for instance, in analyzing "spreads" (how many elements does it take to generate an ideal which gives the closure of a given ideal?) and extending the Briançon–Skoda Theorem.

SARA FARIDI, Dalhousie University, Halifax, NS, B3J 3J5
On ideals of conjugacy classes of nilpotent matrices

We discuss ideals associated to the closure of conjugacy classes of nilpotent matrices. These ideals are indexed by partitions of the size of the matrix. We then restrict to the intersection of the conjugacy class with diagonal matrices, and use a well-known generating set by Tanisaki to produce a new and smaller generating set for these ideals. We also find a minimal generating set in the case of hook partitions, which enables us to easily compute the minimal free resolution.

This is joint work with Riccardo Biagioli and Mercedes Rosas.

NOAM HORWITZ, Cornell University, Ithaca, NY 14853
Linear resolutions of edge ideals

Edge ideals are monomial ideals defined by graphs. We study the minimal free resolutions of such ideals in the case where the resolutions are linear. Explicit resolutions are given under the assumption that the graph associated with the edge ideal satisfies specific combinatorial conditions. Furthermore, we construct a regular cell complex supporting the minimal free resolution in such cases.

COLIN INGALLS, University of New Brunswick, Dept. of Math and Stats
Spaces of Linear Modules on Regular Graded Clifford algebras

The space of regular noncommutative algebras includes regular graded Clifford algebras, which correspond to base point free linear systems of quadrics in dimension n in P^n. The schemes of linear modules for these algebras can be described in terms of this linear system. We show that the space of line modules on a 4-dimensional algebra is an Enriques surface called the Reye congruence, and we extend this result to higher dimensions.

SRIKANTH B. IYENGAR, University of Nebraska, Department of Mathematics, Lincoln, NE 68588
Dimension of the stable category of a commutative ring

Rouquier has introduced a notion of dimension for triangulated categories, and provided estimates for the dimension of stable derived categories of exterior algebras. In my talk, I will discuss similar results pertaining to the stable category of a commutative noetherian ring.

DAVID A. JORGENSEN, University of Texas at Arlington, Arlington, TX 76019, USA
Linear acyclic complexes

In this talk we will investigate the existence of linear acyclic complexes of finitely generated free modules over a commutative local ring with radical cube zero.

ANTONIO LAFACE, Queen's

DIANE MACLAGAN, Rutgers University, Piscataway, NJ 08854, USA
Equations and degenerations of $\overline{M}_{0,n}$

The moduli space $\overline{M}_{0,n}$ has as points all stable genus zero points with n marked points. I will introduce this object, and describe joint work with Angela Gibney (Penn) that gives explict equations for $\overline{M}_{0,n}$ in the Cox ring of a related toric variety. An application of this is an explicit construction of a degeneration of $\overline{M}_{0,n}$ to a toric variety.

CLAUDIA MILLER, Syracuse University, Syracuse, NY 13244
A Riemann–Roch formula for the blow-up of a nonsingular affine scheme

Hilbert polynomials have classically been shown to be determined by intersections with hyperplanes. We give another approach over regular local rings via Intersection Theory on the blow-up scheme. The result comes in the form of a Riemann–Roch formula for the blow-up of a nonsingular affine scheme.

IRENA PEEVA, Cornell University, Ithaca, NY 14853, USA
Generalized Green's Theorem

Green proved Green's Theorem on how the Hilbert function changes after taking a quotient by a generic linear form. He used this result to provide a new and simple proof of Macaulay's Theorem, which characterizes the Hilbert functions of graded ideals in a polynomial ring. Herzog and Popescu extended Green's result to generic forms of any degree, but under the assumption that the ground field has characteristic zero. Later, Gasharov found a new proof that works in all characteristics. We provide a different proof, which works in all characteristics and which works not only over polynomial rings but also yields the new result that the theorem holds over Clements–Lindstrom quotient rings.

This is joint work with Jeff Mermin.

THUY PHAM, University of Toronto at Scarborough
jdeg *of algebraic structures*

Let R be a commutative Noetherian ring and A a finitely generated standard graded R-algebra. We introduce and develop a new degree $\mathrm{jdeg}(\cdot)$ attached to finitely generated graded A-modules. This construction $\mathrm{jdeg}(\cdot)$ coincides with the classical multiplicity $\deg(\cdot)$ when R is an Artinian local ring. It also acquires a global nature in contrast to other extensions of $\deg(\cdot)$ usually requiring R to be local or graded.

An important application of $\mathrm{jdeg}(\cdot)$, which is also the original motivation of this notion, is to measure the length of the chains of graded subalgebras between A and its integral closure $\bar{A}$, constructed by general algorithms. This gives a refinement of recent results to very general graded algebras.

RAVI VAKIL, Stanford University, Stanford, CA 94305
Murphy's Law in algebraic geometry: badly-behaved moduli spaces

We consider the question: "How bad can the deformation space of an object be?" (Alternatively: "What singularities can appear on a moduli space?") The answer seems to be: "Unless there is some *a priori* reason otherwise, the deformation space can be arbitrarily ugly." Hence many of the most important moduli spaces in algebraic geometry are arbitrarily singular, justifying a philosophy of Mumford.

More precisely, every singularity of finite type over $\mathbb{Z}$ (up to smooth parameters) appears on the Hilbert scheme of curves in projective space, and the moduli spaces of: smooth projective general-type surfaces (or higher-dimensional varieties), plane curves with nodes and cusps, stable sheaves, isolated threefold singularities, and more. The objects themselves are not pathological, and are in fact as nice as can be: the curves are smooth, the surfaces have very ample canonical bundle, the stable sheaves are torsion of rank 1, the singularities are normal and Cohen–Macaulay, *etc.*

Thus one can construct a smooth curve in projective space whose deformation space has any specified number of components, each with any specified singularity type, with any specified non-reduced behaviour along various associated subschemes. Similarly one can give a surface over $\mathbb{F}_p$ that lifts to p^7 but not p^8. (Of course the results hold in the holomorphic category as well.)

ADAM VAN TUYL, Lakehead University, Thunder Bay, ON, P7B 5E1
Some resolutions of double points in $\mathbb{P}^1 \times \mathbb{P}^1$

Let Z be a finite set of double points in $\mathbb{P}^1 \times \mathbb{P}^1$ and suppose further that X, the support of Z, is arithmetically Cohen–Macaulay (ACM). I will present an algorithm, which depends only upon a combinatorial description of X, for the bigraded Betti numbers of I_Z, the defining ideal of Z.

This is joint work with Elena Guardo of Catania.

MAURICIO VELASCO, Cornell University, Ithaca, NY
Grobner bases, monomial group actions and the Cox rings of Del Pezzo surfaces

We introduce the notion of monomial group action and study some of its consequences for Gröbner basis theory. As an application we prove a conjecture of V. Batyrev and O. Popov describing the Cox rings of Del Pezzo surfaces (of degree ≥ 3) as quotients of a polynomial ring by an ideal generated by quadrics.

The results presented in this talk are joint work with Mike Stillman and Damiano Testa.

ALEXANDER YONG, University of Minnesota and the Fields Institute
A combinatorial rule for (co)minuscule Schubert calculus

We prove a root system uniform, concise and positive combinatorial rule for Schubert calculus of *minuscule* and *cominuscule* flag manifolds G/P (the latter are also known as *compact Hermitian symmetric spaces*). We connect this geometry to the poset combinatorics of [Proctor '04], thereby giving a generalization of the [Schützenberger '77] jeu de taquin formulation of the Littlewood–Richardson rule, which computes the intersection numbers of Grassmannian Schubert varieties. Our proof introduces *cominuscule recursions*, a general technique to relate the numbers for different Lie types.

I will also briefly discuss connections of the rule to (geometric) representation theory, specifically to Kostant's study of Lie algebra cohomology, and separately, the geometric Satake correspondence of Ginzburg, Mirković–Vilonen *et al.*

This is based on joint work with Hugh Thomas; see math.AG/0608276.