A model for cleaning a graph with brushes was recently introduced. We consider the maximum number of brushes that can be used to clean $d$-regular graphs in this model, focusing on the asymptotic number for random $d$-regular graphs. Various lower and upper bounds are proposed. To get an asymptotically almost sure lower bound we use a degree-greedy algorithm to clean a random $d$-regular graph on $n$ vertices (with $dn$ even) and analyze it using the differential equations method to find the (asymptotic) number of brushes needed to clean a random $d$-regular graph using this algorithm (for fixed $d$).

We further show that for any $d$-regular graph on $n$ vertices there is a cleaning sequence such at least $n(d + 1)/4$ brushes are needed to clean a graph using this sequence. For an asymptotically almost sure upper bound, the pairing model is used to show that at most $n(d + 2\sqrt{d\ln 2})/4$ brushes can be used when a random $d$-regular graph is cleaned. This implies that for fixed large $d$, the maximum possible number of brushes that can be used to clean a random $d$-regular graph on $n$ vertices is asymptotically almost surely $\frac{n}{4}(d + O(\sqrt{d}))$. 

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Cleaning random $d$-regular graphs with brooms