

Rapport du groupe de travail 2a

Créer un curriculum qui allie maîtrise technique et compréhension conceptuelle

Creating a Curriculum that Affords Learners the Opportunity to

Develop Powerful Mathematics

Créer un curriculum qui allie maîtrise technique et compréhension conceptuelle

Comme en témoignent de récentes initiatives de « retour à la base » et d'« enseignement axé sur la compréhension », la question des mathématiques au primaire et au secondaire évolue en grande partie autour de l'hypothèse d'une tension entre la compétence (technique) et la compréhension des concepts. Cette tension est-elle nécessaire? Sinon, est-il possible de créer un programme où la compétence et la compréhension seraient complémentaires (et interdépendantes)? Quel type de ressources et de préparation faudrait-il offrir aux enseignants pour leur permettre de bien présenter un tel programme?

Voilà les questions sur lesquelles s'est penché ce groupe de travail

Creating a Curriculum that Affords Learners the Opportunity to Develop Powerful Mathematics

As evidenced by recent “back-to-basics” and “teaching-for-understanding” movements, much of the debate about grade school mathematics curricula is organized around the assumption that there is a tension between technical proficiency and conceptual understanding. Is this tension a necessary one? Or is it possible to create a curriculum in which proficiency and understanding are framed in terms of complementary—indeed, codependent—relationship? What sorts of resources and preparations would be needed to ensure the successful introduction of such a curriculum?

These are the questions we explored in this working group.

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First working group session

In order to initiate the reflection on the theme, we submit to the group some introductory questions:

- Do you see a difference between proficiency and competence?
- What is meant by mathematical proficiency?
- What kind of relation do you see between technical proficiency and conceptual understanding?
 - Are they opposed?
 - Does one must come before the other?
 - Do they develop together?
- Do you feel some gap between the prescription or intention presented in your curriculum and the way it is actualised in classroom?
- Does mathematical proficiency differ for different grade levels?
- May assessment put some pressure on teaching so that technical proficiency may be favoured in detriment of conceptual understanding?
- How do we assess mathematical proficiency?

A well known card game problem (Wason & Johnson-Laird, 1972).



All those cards have a letter on one side and a number on the other. Which of these four cards is it necessary to turn over to check if the following rule is respected: « If there is an A on one side, so there is a 4 on the other »

Even though one knows the math necessary to solve this problem, it won't be surprising if he gives the wrong answer.

- How does this problem question the dynamic between technical proficiency and conceptual understanding?

Plenary group discussion on Proficiency

With those questions as a starting point, participants did express themselves on different aspects of proficiency. We have group their reflections into four main categories: Teacher proficiency, Student proficiency, Assessment, Curriculum.

Teacher proficiency

- In service teachers: fear or discomfort with math

- There is a fear from elementary schools teachers about mathematics and therefore they may be teaching outside of their comfort zone.
- In Ontario teachers up to grade 8 are arriving as generalists and may not have a strong background in mathematics.
- Why are teachers scared to make mistakes in mathematics and not mistakes in other subjects?
- Fear on maths is on both sides: teachers and students.

- Math for pre service teachers

- Acadia offers a course on high school math to help prepare students for being teachers. The question is can students overcome the deficiencies in mathematics to prepare them to teach.
- At Acadia it is rare individuals who can take a remedial course and then pass calculus.
- At Queens students are asked to take a course that contains junior high mathematics to prepare them to teach it.
- Primary school trainees are very creative in all subject matter but not necessarily in mathematics. Why is this?
- In order for students to develop a proficiency in mathematics the teachers must also have a proficiency in mathematics.

Student proficiency

- Procedural VS conceptual

- Students often do well in the computational knowledge early on but a lack of conceptual understanding has an impact later on when students can not remember the procedural knowledge. Students do well at the technical proficiency early on and then struggle later.
- There is a basic language skill that is necessary for mathematics and students need to learn the language of mathematics. Students need some skills to develop conceptual understanding.
- If students arrive at post-secondary and do not have conceptual understanding of mathematics then they do not want to learn it.
- Chinese students arriving at Acadia do very well at calculus but once they get into later courses they suffer from not having the conceptual understanding.
- If students have the conceptual knowledge they may be blocked from understanding higher level mathematics because they have a lack of techniques.
- What makes people scared of mathematics is the possibility of immediate failure. In other subjects there is a belief that discourse is part of the learning. This is not always the case

in mathematics. There needs to be an understanding that conceptual understanding is important and that mathematics is about discovery.

- Should procedural skills should precede or follow conceptual understanding or are they intimately linked?
- Modelling vs skills?
- Can students use mathematics in day-to-day context?

- Personal backgrounds or individual factors

- Students arrive with diverse backgrounds but there are a lot of problems that are not related to curriculum but to home situations. It is important to create contexts for the students in the curriculum which students can relate to.
- Conceptual understanding and procedural understanding are part of one hand and emotional understanding is the other hand and they have to go together.

- Involvement, Motivation

- Students are not involved in mathematics and therefore have trouble with mathematics.
- Making connections for students with data and contexts that interests them may help to motivate them to do mathematics.
- It is important that a student learn prerequisite knowledge before going on to new topics. This helps create motivation and helps them set higher standards.

Assessment

- In Ontario social promotion may be an issue as students pass each year until they reach high school and then they struggle.
- Statistics show that there is no difference in the success of students who are held back and those that are promoted.
- The PISA study shows that Canada is doing very well internationally and we can not overlook this.
- There are a number of factors that affect how well the students are doing such as assessments, professional development, resources, teacher preparation, etc.
- With the same curriculum and the same resources, very different results can be obtained. Why? Does assessment play a role?

Curriculum

- What do we expect from a mathematics curriculum?
- Do we expect kids to do more mathematics or more kids to do mathematics?
- In Quebec some material was removed to make sure that students could make links and understand the mathematics better.
- Removing grade 13 has had an impact on the maturity of students.
- Intended curriculum vs taught curriculum: There may be big differences. Does assessment play a role? Teacher training? Other?
- Curriculum is meant for students, but also for teachers.
- What are our real expectations from a curriculum?

Second working group session

The working group choose to pursue his reflection upon curriculum matter into 3 sub-groups.

What do we expect from a mathematics curriculum?

Sub-group 1

- Most of the items speak to the taught curriculum and not to the provincial/territorial documents.
- Student engagement
 - Language should use verbs instead of outcomes
 - Fun and challenging
 - Conversations and discussions
 - Inclusive! Success oriented (If there is something students can not do in a compulsory course, you do not do it – room to do something else)
 - Meaningful context – teacher decides
 - Variety – sometimes math is beautiful and meaningful
 - Sensual – Math summarizes a variety of sensory, emotional and tactile experiences
- Appropriate sequencing
 - Developmentally appropriate
 - Prepares the mind for future development
 - Foster maturity, persistence and independence
- Promotes productivity (Danger that these become a goal or tools to obedience instead of for the learning)
 - Work habits – getting the job done
 - Disciplined thinking
 - Orderly habits of working
- Assessment *follows* curriculum, but does not drive it

Sub-group 2:

- *Making sense for understanding* (title of presentation)
- Opening to various teaching and learning strategies
- Co-development of conceptual understanding and procedural knowledge
- Linking general and particular.
- Avoid “déjà vu” feeling due to repetition of similar problems from year to year.
- Present challenges, trying to introduce cognitive conflict
- Deepen meaning of mathematical contents
- Building new knowledge through sharing (community)
- Award importance to mathematical communication (words, symbolism)
- Allow different strategies (do not ask for uniformity)
- Demystification of math and valorisation
- Various evaluation strategies and evaluation as help for learning
- Encourage different means of communication
- Encourage creativity (imagination, abstraction, ...)

Sub-group 3

- Conceptual understanding of mathematics topics
 - Should be able to derive rather than memorize
 - Do not do recipe math
- Use and understand “techniques”
- Sequenced logically
- Basic number sense, spatial sense, “numeracy”, “mathematical literacy”
- Connections within mathematics and to other areas
- Develop problem solving and thinking skills
- Manageable in the amount of time “given”
- Clarity of outcomes in terms of interpretations, depth and breadth (i.e. clarity of expectations)
- Consistency of wording and definition
- Works on the intuitive mathematical abilities of students

Some other issues

- Teachers knowledge of mathematics content and pedagogy
- There are fewer mathematics teachers with a degree in mathematics
- Assessment can drive teaching
- Do teacher preparation programs prepare teachers to teach math
- Availability of resources
- Societal issues and attitudes

Discussion:

- About relevance of mathematics

- There is a need for students to see relevance in mathematics to all aspects of their life. Show where mathematics is really used in our lives.
- Understanding mathematics is based on extracting it from the world around us (i.e. models)
- Making mathematics interesting is an important part of the mathematics curriculum.
- Some mathematicians may feel that it is not real mathematics until it is “pure.”
- Many tools that are learned in mathematics are tools to be used for a greater goal.
- It is important to view problems mathematically.
- In Québec there are 5 domains that students are asked to incorporate in all of their subjects.
- What is meant by the word “real life” and does it have more relevance at the elementary level and not at the high school level.
- Connections need to be made from mathematics to other areas and within mathematics.

- “Back to basics” and “conceptual understanding” need to exist together. There is also an aspect of the affective domain that is also important for students.

- People at this conference have an opportunity to set a mandate toward mathematics much like was done in English over the past 30 years in moving the focus toward grammar.

- This is an opportunity to make a bold statement about mathematics curriculum.

Third and final session

After we have discussed what we expected from a math curriculum, we did address our 3 first questions on the last working group session. Here are the propositions to which the group came at the end of the work sessions.

1) Is there a tension between technical proficiency and conceptual understanding and is it necessary?

When the written curriculum is considered, the perceived tension between technical proficiency and conceptual understanding is artificial. This tension does however, exist when the taught and learned curricula are considered and can be a catalyst for change and progress.

2) Is it possible to deliver a curriculum in which proficiency and understanding are framed in terms of a complementary – indeed, codependent - relationship?

A good curriculum focuses on the big ideas and is guided by a teacher with a profound understanding of fundamental mathematics. This curriculum would be problem-based; it would relate mathematics to other areas of life; it would value communication of mathematical ideas; it would be delivered with attention to the readiness of students; it would challenge them in imaginative ways; and it would be aware of longitudinal development of mathematical thinking.

3) What sort of resources and preparations would be needed to ensure the successful introduction of such curriculum?

Preparation

- Pre-service programs need more courses (and time spent) on what to teach and how to teach in math and connected to what they will teach.
- On going professional development for teachers.

School

- Teaching assignments must reflect interest and competency (middle school, Junior/Senior High school)
- Upper elementary (grades 5-8) teachers could teach math only

Resources

- Support at grassroots and partnerships with post-secondary institutions
- Mechanisms for sharing experiences within the mathematics education community
- Text and technical resources that support curriculum (e.g. copies of students work.

Reference

WASON, P.C. & JOHNSON-LAIRD, P.N. (1972). *Psychology of Reasoning. Structure and Content*. Boston, Harvard University Press.

Voici les propositions, en réponses aux trois questions initiales, auxquelles le groupe est arrivé à l'issue des sessions de travail.

1) Y a-t-il une tension entre la compétence (technique) et la compréhension des concepts. Cette tension est-elle nécessaire?

La tension perçue entre maîtrise technique et compréhension conceptuelle est artificielle lorsqu'on regarde le curriculum écrit. Toutefois, cette tension existe lorsque l'on considère le curriculum enseigné et celui appris. Une telle tension peut alors être un déclencheur, un ressort pour initier un changement et amener un progrès.

2) Est-il possible de créer un programme où la compétence et la compréhension seraient complémentaires (et interdépendantes)?

Un bon curriculum se centre sur les idées principales et est piloté par un enseignant ayant une compréhension profonde des mathématiques. Un tel curriculum s'appuie sur la résolution de problèmes, il relie les mathématiques aux différents domaines de la vie, il valorise et favorise la communication des idées mathématiques, il prend en compte la préparation des élèves, il pose des défis originaux aux élèves et sollicite leur imagination, il prend en compte le développement de la pensée mathématique.

3) Quel type de ressources et de préparation faudrait-il offrir aux enseignants pour leur permettre de bien présenter un tel programme?

Formation des maîtres

- La formation initiale requiert plus de cours (et plus de temps) sur ce qui doit être enseigné et sur la façon de le faire, et ce, en lien avec les mathématiques que les futurs enseignants auront à enseigner.
- Il faut assurer une formation continue et alimenter le développement professionnel.

À l'école

- Les tâches des enseignants devraient refléter leurs intérêts et leurs compétences
- À « l'upper elementary » (5^e à 8^e années) les enseignants pourraient enseigner uniquement les mathématiques.

Ressources

- Offrir un support à la base, aux enseignants, et favoriser des partenariats avec les institutions post-secondaires.
- Prévoir des mécanismes pour partager ces expériences de partenariat.
- Offrir des ressources techniques et des textes, par exemple des productions d'élèves analysées.